A Survey on Algorithmic Approaches for Solving Tourist Trip Design Problems

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Abstract

The tourist trip design problem (TTDP) refers to a route-planning problem for tourists interested in visiting multiple points of interest (POIs). TTDP solvers derive daily tourist tours, i.e., ordered visits to POIs, which respect tourist constraints and POIs attributes. The main objective of the problem discussed is to select POIs that match tourist preferences, thereby maximizing tourist satisfaction, while taking into account a multitude of parameters and constraints (e.g., distances among POIs, visiting time required for each POI, POIs visiting days/hours, entrance fees, weather conditions) and respecting the time available for sightseeing on a daily basis. The aim of this work is to survey models, algorithmic approaches and methodologies concerning tourist trip design problems. Recent approaches are examined, focusing on problem models that best capture a multitude of realistic POIs attributes and user constraints; further, several interesting TTDP variants are investigated. Open issues and promising prospects in tourist trip planning research are also discussed.

1 Introduction

Tourists that visit a destination for one or several days, are facing the problem to decide which points of interest (POIs) would be more interesting to visit and to determine a route for each trip day, i.e., which POIs to visit as well as the visiting order among them. This is a challenging quest that involves a number of constraints such as the visiting time required for each POI, the POI's visiting days/hours, the traveling distance among POIs, the time available for sightseeing on a daily basis and the "degree of satisfaction" (termed "profit") associated with the visit to each POI (based on personal profile and preferences).

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Personalized Electronic Tourist guides (PETs) may be used to derive personalized tourist routes [36], [60], [80], [81], [95]. Based on a list of personal interests and preferences, up-to-date information for the POIs and information about the visit (e.g. date of arrival and departure, accommodation address, etc), a PET can suggest feasible and near-optimal routes that include visits to a series of most interesting POIs [131]. PETs featuring route planning services, typically offer three main functionalities [59]: (i) The *recommendation* functionality which generates a list of POIs for each different tourist profile; each POI in the list is associated with a profit and a visit duration. (ii) The *route generation* functionality which employs an algorithm to generate personalized tourist routes using the list of POIs generated by the recommendation functionality as well as other tourist-related data (e.g., days of visit, duration of the routes), POI-related data (e.g., opening hours) and transportation data (e.g., public transport network information). (iii) The *customization* functionality which allows tourists to modify the generated personalized route (add/remove or reorder POIs) to better fit their needs.

A number of web and mobile applications have recently incorporated tourist route recommendations within their core functionality [130], [98], [63]. In effect, they incorporate the main functionalities of PETS i.e., they generate personalized routes (see e.g., the City Trip Planner [39], the mtrip [98]) taking into account several user-defined parameters within their recommendation logic (days of visit, preferences upon POI categories, start/end location, visiting pace/intensity), while also allowing the user to manually edit the derived routes, e.g. add/remove POIs. Recommended tours are visualized on maps [39], [98], [63], allowing users to browse informative content on selected POIs. Some tools also offer augmented reality views of recommended attractions (e.g., [98]).

The generic problem of personalized tourist route generation which is mainly associated with the route generation functionality of mobile tourist guides and PETs, has been defined as the "Tourist Trip Design Problem" (TTDP) [131]. The modeling of the TTDP is approached considering the following input data (see Figure 1):

- A set of candidate POIs, each associated with a number of attributes (e.g. type, location, opening days/hours, etc).
- The travel time among POIs calculated using multi-modal routing information among POIs, i.e. tourists are assumed to use all modes of transport available at the tourist destination, including public transportation, walking and/or bicycle.
- The "profit" of each POI, calculated as a weighted function of the objective and subjective importance of each POI (subjectivity refers to the users' individual preferences and interests on specific POI categories).
- The number of routes that must be generated, based upon the period of stay of the user at the tourist destination.
- The anticipated visiting duration of a user at a POI which derives from the average duration and the user's potential interest for that particular POI.
- The daily time limit T that a tourist wishes to spend on visiting sights; the overall daily route duration (i.e. the sum of visiting times plus the overall time spent moving from a POI to another which is a function of the topological distance) should be kept below T.

By solving the TTDP we expect to derive daily, ordered visits to POIs, while respecting user constraints related to the travel cost and POI attributes. High quality TTDP solutions should feature POI recommendations that match tourist preferences, i.e., maximize the collected profit,

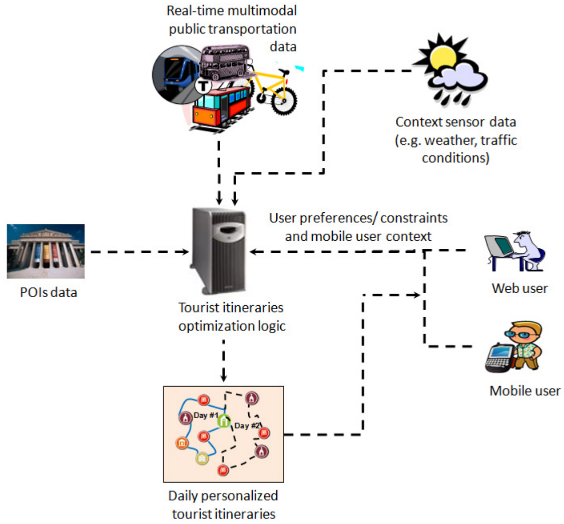

Figure 1: Input data and recommended itineraries in TTDP.

and near-optimal feasible route scheduling. Note that a number of different variants of the TTDP may be defined by considering different parameters and constraints of the above generic problem.

The literature relevant to the TTDP covers a broad range of topics, including systems (e.g. [130], [138]), architectural (e.g. [136]), evaluation (e.g. [61]) and algorithmic (e.g. [62], [129]) perspectives. The algorithmic approaches for solving TTDP variants represents the most crowded field of research among them and represents the main focus of this survey.

Outlook of algorithmic approaches for solving the TTDP. A basic classification of the TTDP variants may be based on the number of the derived routes as follows: (i) single tour TTDP variants aiming at finding a single tour that maximizes the collected profit while respecting certain tourist constraints and POI attributes, and (ii) multiple tour TTDP variants aiming at finding multiple tours based upon the number of days the tourist's visit will last.

Single tour variants of the TTDP can be modeled using single-criterion variants of the Travelling Salesman Problem with Profits (TSPP), a bicriteria generalization of TSP with two conflicting objectives: maximizing the collected profit and minimizing the travel cost [79]. The Orienteering Problem (OP) [123] is a single-criterion variant of TSPP which seeks for a tour that maximizes the total collected profit while maintaining the travel cost under a given value, i.e., the travel cost objective is turned to a constraint. Clearly, the OP may be used to model the simplest version of the TTDP wherein the POIs are associated with a profit (i.e. user satisfaction) and the goal is to find a single tour that maximizes the profit collected within a given time budget (time allowed for sightseeing in a single day). Extensions of the OP have been successfully applied to model more complex versions of the single tour TTDP. The OP with Time Windows (OPTW) considers visits to locations within a predefined time window (this allows modeling opening and closing hours of POIs). The Time-Dependent OP (TDOP) considers time dependency in the estimation of time required to move from one location to another and therefore, it is suitable for modeling multi-modal transports among POIs.

The extension of TSPP to multiple tours is known as Vehicle Routing Problem with Profits (VRPP) [10]. One known variant of the VRPP is the Team Orienteering Problem (TOP) [30], i.e., the extension of the OP to multiple tours. The TOP as well as its extensions i.e., the

TOP with Time Windows (TOPTW) and the Time-Dependent TOPTW (TDTOPTW), have been used to model different versions of the multiple tour TTDP. Several further extensions of the TOP have been proposed that allow the modeling of even more complex versions of the multiple tour TTDP, e.g. the Multi-Constraint Team Orienteering Problem with Time Windows (MCTOPTW) takes into account multiple user constraints such as the overall budget that may be spent for POI entrance fees.

Contributions and structure of the article. The main body of the algorithmic and operational research literature dealing with TTDP modeling and solving focuses on OP, TOP as well as their extensions and variants. Inevitably, the vast majority of the algorithmic approaches presented in this survey concern these problems. Specifically, we survey exact, approximate and heuristic approaches for solving optimization problems that are employed for modeling different versions of the TTDP. This survey goes far beyond than just bringing together anything relevant to TTDP. The main contributions of our work are the following: (a) focusing on TTDP, our survey approach is towards incremental generalizations of the basic OP problem which better capture TTDP modeling requirements; (b) we provide comparative analysis among alternative approaches for each specific problem (variants of OP); (c) in addition to OP variants, we survey a multitude of relevant problems, which could also capture various aspects and modeling parameters of the TTDP, explaining their utility in addressing TTDP requirements; (d) we detail open research issues and suggest promising directions for future work with respect to algorithmic approaches to the TTDP.

The remainder of this article is structured as follows: Section 2 presents algorithmic approaches for solving single tour variants of the TTDP, i.e. problems aiming at finding a single tour that maximizes the profit under certain constraints such as the OP, the OPTW, the TDOP as well as variants of OP. Section 3 surveys algorithmic approaches dealing with multiple tour variants of the TTDP such as the TOP, the TOPTW, the TDTOPTW and variants of the TOP. It is noted that particular emphasis is given to algorithmic techniques for solving problems highly relevant to more complex and realistic versions of the TTDP (e.g. TOPTW and TDTOPTW). Section 4 highlights combinatorial problems that may be used for modeling variants of the TTDP which have not been widely studied in the literature, and surveys algorithmic approaches dealing with such problems. Although diverging from a 'strict' definition of the TTDP, those approaches offer intuitions for a more versatile analysis of the problem and reveal promising prospects to extend the functionality of existing TTDP solvers. Finally, Section 5 concludes the paper providing new prospects in tourist route planning research. Specifically, we discuss (i) quality improvements upon existing solution approaches, (ii) modeling TOPTW generalizations, (iii) modeling problems relevant to TTDP and (iv) employing parallel computing techniques to design new heuristics for the TTDP.

2 Single tour TTDP solution approaches

Single tour variants of the TTDP can be modeled using single-criterion variants of the Travelling Salesman Problem with Profits (TSPP), a bicriteria generalization of TSP. Specifically, in TSPP we are given a network in which nodes are associated with profits and links with travel costs, and the goal is to find a tour (which starts and ends at a specified node - the depot) over a subset of nodes such that the collected profit is maximized while the travel cost is minimized. The problem was introduced under the name multiobjective vending problem in [79]. In [20] the first exact Pareto fronts (sets of non-dominated solutions) have been given for TSPP instances obtained from classical TSP instances, available in the TSPLIB [105]. In [75] a hybrid meta-heuristic was presented that yields high-quality approximations of the efficient frontier for TSPP.

There are three single-criterion variants of TSPP based on how the two objectives of maximizing the collected profit and minimizing the travel cost are handled:

(i) The Profitable Tour Problem (PTP) introduced in [47], searches for a tour that maximizes the collected profit minus the travel cost, i.e., the two objectives are combined in one objective function.

(ii) The Prize Collecting TSP (PCTSP) introduced in [18] aims at finding a tour that minimizes the travel cost, with the total tour profit being not smaller than a given value, i.e., the profit objective is stated as a constraint.

(iii) The OP seeks for a tour that maximizes the total collected profit while maintaining the travel cost under a given value, i.e., the travel cost objective is stated as a constraint.

TSP is a special case of both PTP and PCTSP and, therefore, the two problems belong to the class of NP-hard problems. Bienstock et al. [21] developed the first approximation algorithm for PTP with a performance guarantee bound of 5/2. This bound was improved in [68] where a $2 - 1/(n-1)$-approximation algorithm was given, where n is the number of nodes. Awerbuch et al. [17] gave an approximation algorithm for the PCTSP based on an approximation algorithm for the k-minimum-spanning-tree problem ([15]). There also exists literature on exact, heuristic and metaheuristic algorithms for PTP and PCTSP as well as variants of these problems (see [54] for a survey).

The OP more closely formulates the single tour version of the TTDP than the other two single-criterion TSPP variants. The vast majority of the papers in the TTDP literature use the OP and its extensions to model different variants of the problem. Therefore, in the sequel of this section we focus on OP, its extensions and variants (see Figure 2) and survey algorithmic approaches for solving them.

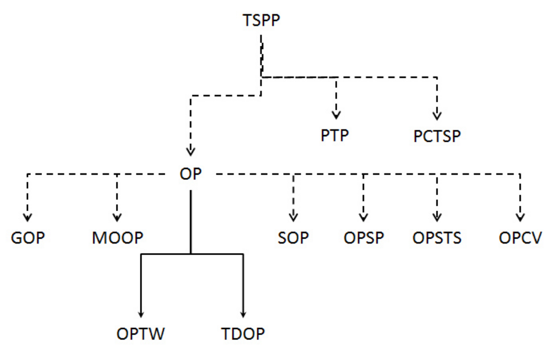

Figure 2: Single-tour TTDP solution approaches (solid arrows denote problem extensions, while dashed arrows denote variants).

2.1 Orienteering Problem (OP)

The OP was introduced by Tsiligirides [123] named after a sport game called orienteering. Other names used for the OP are Selective Traveling Salesperson Problem (STSP) [87], Maximum Collection Problem (MCP) [77] and Bank Robber Problem [13]. OP can be formulated as follows: Let $G = (V, E)$ be an edge-weighted graph with profits (rewards or scores) on its nodes.

Given a starting node s, a terminal node t and a positive time limit (budget) B, the goal is to find a path from s to t (or tour if $s = t$) with length at most B such that the total profit of the visited nodes is maximized (see Figure 3).

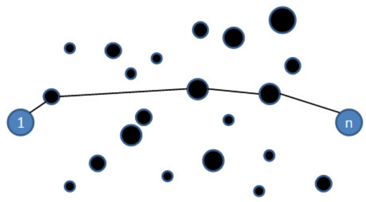

Figure 3: OP illustration. Circles' radius denote nodes' profit.

OP can be formulated as an integer programming problem as follows [127]: Let N be the number of nodes labeled by $1, 2, \ldots, N$ where $s = 1$ and $t = N$, p_i be the profit of visiting node i and c_{ij} be cost of traveling from i to j. For every path from 1 to N, if node i is followed by node j we set the variable x_{ij} equal to 1 or equal to 0 otherwise. Finally, u_i denotes the place of node i in the path. With this notation we have the following relations:

$$\max \sum_{i=2}^{N-1} \sum_{j=2}^{N} p_i x_{ij}, \tag{1}$$

s.t.

$$\sum_{j=2}^{N} x_{1j} = \sum_{i=1}^{N-1} x_{iN} = 1, \tag{2}$$

$$\sum_{i=1}^{N-1} x_{ir} = \sum_{j=2}^{N} x_{rj} \leq 1, \text{ for all } r = 2, \ldots, N-1, \tag{3}$$

$$\sum_{i=1}^{N-1} \sum_{j=2}^{N} c_{ij} x_{ij} \leq B, \tag{4}$$

$$2 \leq u_i \leq N, \text{ for all } i = 1, 2, \ldots, N, \tag{5}$$

$$u_i - u_j + 1 \leq (N-1)(1 - x_{ij}), \text{ for all } i, j = 2, \ldots, N, \tag{6}$$

$$x_{ij} \in \{0, 1\}, \text{ for all } i, j = 1, \ldots, N. \tag{7}$$

The objective function (1) is to maximize the total profit of visited nodes. Constraint (2) ensures that the path starts at node 1 and ends at node N. Constraint (3) ensures that the path starting at node 1 and ending at node N is connected and each node is visited at most once. Constraint (4) ensures that the path meets the time budget. Finally, constraints (5) and (6) ensure that there are no subtours.

The most important types of OP considered so far depend on whether the graph is undirected [123, 19] or directed (directed OP) [100], whether there is no fixed terminal node but only a fixed starting node called root (rooted OP) [13, 34] or whether there are not fixed end points at all (unrooted OP) [67] and their combinations. OP is harder than rooted OP, which in turn is harder than unrooted OP, since algorithms for rooted OP can be used to solve unrooted OP by considering each node of the graph as the root. Likewise, OP can be used to solve rooted OP by considering as the starting node the root and each node of the graph as the finishing node.

OP is NP-hard (e.g. see [70], [87]). Hence, exact solutions for the OP are only feasible for graphs with a small number of nodes. Some of the exact algorithms proposed for the OP are based on branch-and-bound [87, 104] and branch-and-cut [66, 56]. There exist a number of approximation algorithms for the above variants of OP, which either are rather difficult to implement or have high-order polynomial computational complexity and hence high execution time in practice. Still, the study of approximation algorithms offers valuable insights over the inherent difficulties of the problem at hand. Moreover, some of approximation design techniques may as well be used, after necessary simplification, for developing effective heuristic solutions in place of other ad-hoc solutions that most general metaheuristic methodologies commonly derive.

Some helpful remarks concerning the approximability of certain OP variants are the following:

- The rooted OP is an APX-hard problem [1] (e.g. see [23], where it is proven that rooted OP is NP-hard to approximate to within a factor of $\frac{1481}{1480}$).
- In the approximation algorithms for the OP, the input graph can be restricted to graphs having nodes with unit profit since Korula [82, Lemma 2.6] proved that an a-approximation algorithm for the OP with unit profits yields an $a(1+O(1))$-approximation algorithm for OP with arbitary profits. The basic idea is to use a standard scaling technique to adjust the weights into integers from 1 to n^2, where n is the number of nodes, and then to transform the graph to a new graph with at most n^3 nodes having unit profits. A solution with the above approximation is derived for the initial instance of OP by applying an a-approximation algorithm on the newly transformed graph.
- An approach for approximating the unrooted OP in undirected graphs comes from approximation algorithms for the k-TSP problem (find a tour of minimal length while visiting at least k nodes). The basic idea is to break such a tour into pieces bounded by B and then pick the one with the largest profit (for more details, see [17]).
- Usually, the approximation algorithms for the OP have higher complexity in directed graphs than in undirected graphs (e.g. see [31]).

One of the first works for approximating the rooted OP is that of Arkin et al. [13] that gives a $(2+\epsilon)$-approximation algorithm for the OP restricted to points in the 2-dimensional plane. The fundamental idea to approximate the rooted OP in undirected graphs was presented by Blum et al. in [22], [23]. They use, as an intermediate step, the solution of the min-excess $(s-t)$ path problem (find a minimum-excess [2] path connecting fixed nodes s and t that visits at least k nodes or collecting at least k profit). The basic idea is to guess[3] the profit P_{OPT} of the optimal solution of the rooted OP and try to compute for every node the min-excess path from the root to the node that collects at least a fixed fraction of P_{OPT}, until a path is found with length at most B. In this work they obtain a 4-approximation algorithm for rooted OP in undirected graphs by using a $(2+\epsilon)$-approximation to the min-excess $(s-t)$ path problem. In fact, most subsequent approximation algorithms (e.g. see [31]) use the solution of a min-excess path problem as an intermediate step.

Later, Bansal et al. [19] introduce a 3-approximation algorithm for the OP in metric spaces. In their approach they show that a $(2+\epsilon)$-approximation to the min-excess $(s-t)$ path problem can be used to obtain a 3-approximation for the OP, hence, improving the previous result by Blum et al. [22], [23].

Chen et al. [34] present a PTAS for the rooted OP in $\mathbb{R}^d$, where every location has unit profit. In order to create the PTAS, an approximation algorithm is presented for the $k-$TSP in

[1]A problem for which there is a constant c such that it is NP-hard to find an approximation algorithm with approximation ratio better than c.

[2]Excess of an $s-t$ path is the difference of the path length from the shortest $s-t$ path.

[3]Namely, try exhaustive search.

$\mathbb{R}^d$ based on Mitchell's approximation algorithm for the $k-$TSP [96] and Arora's work on the same problem [14].

Chekuri and Pal [33] give an $O(\log n)$-approximation algorithm for solving the OP in directed graphs that runs in quasi-polynomial time. In their formulation of OP, called submodular OP, the total profit of the nodes visited is not necessarily the sum of the profit of each node but has the submodular property, i.e., for subsets A, B of the set of nodes the total weight f satisfies the inequality: $f(A \cup B) \leq f(A) + f(B) - f(A \cap B)$.

Chekuri et al. [31] give approximation algorithms for the OP in directed and undirected graphs. In particular, they give a $(2+\epsilon)$-approximation algorithm for the undirected OP with running time $n^{O(1/\epsilon)}$ and an $O(\log^2 OPT)$ approximation algorithm for directed OP, where OPT denotes the number of nodes in an optimal solution. They follow Blum et al. focusing on the k-stroll problem (i.e. find a minimum length $s-t$ path that visits at least k nodes) and give bi-criteria approximations for k-stroll in directed and undirected graphs with respect to the path length and the number of nodes visited.

Nagarajan and Ravi [100] give an $O(\frac{\log^2 n}{\log\log n})$-approximation algorithm for the OP in directed graphs, by approximately solving a number of problems in the following order: from minimum ratio ATSP to directed k-path problem, then to the minimum excess problem and finally to OP in directed graphs. First, they present a polynomial time $O(\frac{\log^2 n}{\log\log n})$ bi-criteria approximation algorithm for the directed k-TSP problem (find a minimum length tour that contains a specified root and at least other k nodes), by using an $O(\frac{\log^2 n}{\log\log n})$-approximation algorithm for minimum ratio ATSP problem, due to Asadpour et al [16]. They reduce the directed k-path problem to the directed OP. More specifically, they go from directed k-path problem to directed minimum excess problem and finally to OP in directed graphs.

Table 1 summarizes the approximation algorithms for the OP in directed and undirected graphs and their approximation ratio.

Table 1: Approximation algorithms for the OP

Reference	Directed OP	Undirected OP	Approximation Ratio	Time
Blum et al. [22]		✓	4	polynomial
Bansal et al. [19]		✓	3	polynomial
Chekuri et al. [31]		✓	$(2+\epsilon)$	polynomial
Chekuri and Pal [33]	✓		$O(\log n)$	quasi-polynomial
Chekuri et al. [31]	✓		$O(\log^2 OPT)$	polynomial
Nagarajan & Ravi [100]	✓		$O(\frac{\log^2 n}{\log\log n})$	polynomial

For practical applications, many researchers propose heuristics to tackle the OP, based on different approaches. Some representative methods are discussed in the sequel. Tsiligirides [123] presents two algorithms for the OP. A stochastic algorithm based on Monte-Carlo techniques that constructs a large number of routes and picks the one with the maximum profit and a deterministic heuristic algorithm, that partitions the geographic area into concentric circles and restricts the allowed routes into the sectors defined by the circles.

In [70] a center-of-gravity heuristic for the OP is presented where the solution tour is constructed by the cheapest insertion procedure according to a combined measure for node selection. Golden et al. [69] improve the center-of-gravity heuristic by rewarding nodes associated with above-average tours while penalizing those associated with below-average tours.

In [103] Ramesh et al. propose a four-phase heuristic. After choosing the best solution from iterations over a set of three phases (node insertion, edge exchange and node deletion), a fourth phase is entered, where one attempts to insert unvisited nodes into the tour.

In [134] the authors apply a neural network approach to solve the OP. They derive an energy function and learning algorithm for a modified, continuous Hopfield neural network.

Chao et al. [29] propose a heuristic algorithm for the OP that proceeds as follows. Initially, the set of nodes is partitioned in a greedy way into paths each with length bounded by B and the current solution is the path with the highest profit. Then an iterative method is employed. At each iteration a local search procedure is applied to improve the current solution. However, if a better solution is not found, a solution with slightly less profit is accepted. At the end of the iteration a perturbation move is applied, wherein a number of nodes (that depends on the current iteration) with the smallest ratio of profit to insertion cost are removed from the solution.

In [67] a tabu search heuristic for the unrooted OP is presented. The algorithm iteratively inserts clusters of nodes in the current tour or removes a chain of nodes. Compared to the previous approaches, this method reduces the chance to get trapped in a local optimum. Tests performed by the authors on randomly generated instances with up to 300 nodes show that the algorithm yields near-optimal solutions.

2.2 Orienteering Problem with Time Windows (OPTW)

In OP with Time Windows (OPTW) each node of the graph G can be visited only within one or more specific time intervals (windows) which may be different for each node (see Figure 4). Vansteenwegen et al. [127] argue that time windows significantly affect the nature of OP and its respective algorithmic approaches. For instance, reducing the travel time by reordering scheduled visits, is no longer appropriate due to the time windows. Actually, it has been proved that OPTW is NP-hard even on the line [124].

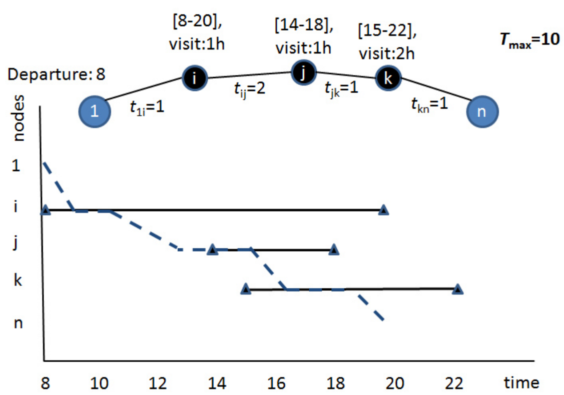

Figure 4: OPTW illustration (dashed lines denote the scheduled route, while triangles opening/closing times)

Righini and Salani [107] give two exact dynamic programming algorithms for the OPTW. The first algorithm uses bidirectional search and the label of each node u used in the algorithm, is a binary vector representing the nodes included in the path ending at u. In the second method, the state space relaxation (SSR) [38] is applied, where the label is an integer denoting the number of visits along the path. Since in the second method a node may be visited more than once due to the reduced information kept at each label, the authors correct this by applying the decremental

SSR (DSSR) method [106], which is an iterative algorithm optimally solving the relaxed problem with the additional constraint that a specific set of nodes cannot be visited more than once.

Kantor and Rosenwein [76] proposed two heuristics for solving the OPTW. The first, the insertion heuristic, incrementally builds the solution and at each step it selects the node with the highest ratio of profit over insertion cost as the next node to be inserted in the path. The second heuristic, the tree heuristic, is employed when the time windows constraints are tight and the input graph nodes are relatively few. By a depth first search exploration of the input graph, it maintains a number of partial solutions simultaneously and repeatedly inserts new nodes in these partially constructed paths as long as the attempted insertion satisfies the problem constraints and some heuristic criteria quantifying the potential solution improvement yield from this insertion.

Also, a number of OPTW approximation algorithms have been proposed in the literature. Bansal et al. [19] gave a $(3\log^2 n)$-approximation algorithm for the OPTW. The main idea is to partition the nodes into different groups according to their time windows and in such a way that OPTW can be solved in each group ignoring time windows. The final solution is derived by stitching the solutions of these subproblems using a dynamic programming approach.

Chekuri and Kumar [32] gave a 5-approximation algorithm for the OPTW with at most k distinct time windows that runs in time polynomial in $(n\Delta)^k$, where Δ is the maximum distance in the metric space and n is the number of nodes. They utilize an approximation algorithm for the maximum coverage problem with group budget constraints[4] and a 3-approximation algorithm of Bansal et al. [19] for the OP.

Later, Chekuri and Pal [33] gave an $O(\log OPT)$-approximation algorithm for rooted OPTW in directed graphs where the total weight of the nodes visited has the submodular property. Their approach, based on a variant of an algorithm for directed s-t connectivity due to Savitch [108], is recursive and greedy and runs in quasi-polynomial time. An application of this algorithm can be found in [37] where travel itineraries for a city are constructed from information collected in the social breadcrumb *Flickr* about the preferences of tourists visiting the city.

Also, Chekuri et al. [31] inspired by the technique of Bansal et al. [19] proved that an α-approximation algorithm for the OP yields an $O(\alpha \max\{\log OPT, \log L\})$ approximation algorithm for the OPTW in directed (and undirected) graphs, where OPT denotes the number of nodes in an optimal solution and L is the ratio of the longest to the shortest time window.

Finaly, Frederickson et al. [58] proposed approximation algorithms for the travelling repairman problem (TRP) in a metric graph or a tree. TRP is a variant of unrooted OPTW, which aims at visiting the maximum number of customers within their time window while two different customers may share the same location. For the case that all time windows have equal length (i.e. unit length), the algorithm initially trims all time windows into subwindows (of half length) with specific ends. Then, for each trimmed window, a new instance containing only the customers with this trimmed time window is considered and for each (s,t) pair of customers and for each different number of nodes that may exist on the $s-t$ path, the corresponding minimum travel $s-t$ path is determined. Then, solutions of the minimum $s-t$ paths for the different trimmed time windows are combined appropiately using dynamic programming, deriving a feasible solution. This algorithm obtains a 3-approximation ratio with running time $O(n^4)$ when the input graph is a tree and a $(6+\epsilon)$-approximation for a general graph with $n^4 \cdot n^{O(\frac{1}{\epsilon^2})}$ running time. Then, the authors generalize their method for time windows with different lengths and they derive an $O(\log L)$-approximation algorithm where L is the ratio of the maximum to minimum time length of all time windows.

[4]Given an integer k and a collection of subsets, of a set S, partitioned into groups, pick k subsets of that collection such that the cardinality of their union is maximized with the restriction that at most one set is picked from each group.

Table 2: Approximation algorithms for the OPTW

Reference	Undirected	Directed	Approximation Ratio	Time
Bansal et al. [19]	✓		$(3\log^2 n)$	polynomial
Chekuri and Kumar [32]	✓		5	polynomial in $(n\Delta)^k$
Chekuri and Pal [33]		✓	$O(\log OPT)$	quasi-polynomial time
Frederickson et al. [58]	✓		$O(\log L)$	polynomial time

2.3 Time Dependent Orienteering Problem (TDOP)

Time-dependent route planning incorporates time dependency in calculating cost of edges, i.e. travelling times among nodes. Time dependency is useful for modeling transfers among nodes through multimodal public transportation. Time Dependent OP (TDOP) was introduced by Formin and Lingas [57]. TDOP is MAX-SNP-hard since a special case of TDOP (the time-dependent maximum scheduling problem) is MAX-SNP-hard [117]. An exact algorithm for solving TDOP is given by Li et al. [90] using a mixed integer programming model and a pre-node optimal labeling algorithm based on the idea of dynamic programming. Moreover, Li [89] proposes an exact algorithm for TDTOP based again on dynamic programming principles. However, both algorithms are of exponential complexity. Fomin and Lingas [57] give a $(2+\epsilon)$ approximation algorithm for rooted and unrooted TDOP (which runs in polynomial time if the ratio R between the maximum and minimum traveling time between any two sites is constant). When considering unrooted TDOP, its running time is $O\big((2R^2(\frac{2+\epsilon}{\epsilon}))!\frac{2R^2}{\epsilon}n^{2R^2(\frac{2+\epsilon}{\epsilon})+1}\big)$, and for rooted TDOP its running time increases by the multiplicative factor $O(\frac{Rn}{\epsilon})$ (the key idea is derived from Spieksma's algorithm [117] for Job Interval Selection Problem, which employs a divide-and-conquer approach). First, the problem is split in smaller ones. Exact solutions are found to each smaller problem and later combined (stitch) to obtain an approximate solution.

Abbaspour et al. [1] investigated a variant of Time Dependent OP with Time Windows (TDOPTW) in urban areas, where the nodes are partitioned into the POIs (associated with profits and time windows) and multimodal transportation stops which do not have profit. A genetic algorithm is proposed for the problem that uses as a subroutine another genetic algorithm for solving the shortest path problem between POIs.

2.4 Variants of OP

In this subsection we present a number of OP variants that have been introduced in the literature to model different versions of TTDP. Note that these problems are not extensions of the OP that take into account only constraints on the availability of nodes or time dependency in calculating cost of edges. Rather they involve more complex formulations considering for example, multiple objectives or constraints, stochastic aspects of different attributes of the problem, etc.

1. The Generalized Orienteering Problem (GOP), wherein each node of the network is assigned a set of benefit values. For example, in the case of a POI, the benefit values may be related to natural beauty, cultural interest, historical significance, educational interest. The overall objective function may comprise any combination of the different benefits. Nonlinear objective functions make the GOP more difficult to solve than OP. In [134] a heuristic was designed to solve GOP using artificial neural networks, while in [135] a straightforward genetic algorithm was given that yields comparable results. In [110] an iterative algorithm was presented for the problem.

2. The Multi-Objective Orienteering Problem (MOOP) is the multi-objective variant of the OP which was formulated in [109] as follows. Each node (POI) may be assigned to different categories (e.g., culture, history, leisure, shopping) and provide different benefits for each category. The aim of MOOP is to find all Pareto efficient solutions without violating the maximum travel cost restriction. In [109] two metaheuristic solution techniques for the bi-objective OP were presented. The first is an adaptation of the Pareto Ant Colony Optimization metaheuristic developed by Doerner et al. [50]. The second is a multi-objective extension of VNS [72].

3. The following stochastic variants of the OP have been studied in the literature:

   - The Orienteering Problem with Stochastic Profits (OPSP), in which the nodes are associated with normally distributed profits. The problem was introduced in [74] and aims at finding a tour that starts and finishes at the depot, visits a subset of nodes within a time limit, and maximizes the probability of collecting more than a prespecified target profit level. In [74] the authors present an exact solution approach based on a parametric formulation of the problem for solving small problem instances and a Pareto-based bi-objective genetic algorithm for larger instances that is based on the conflict between high mean profit and low variance in a solution.
   - The Stochastic Orienteering Problem (SOP), in which each node is associated with a deterministic profit and a random service time. The visit time of a POI is not known until the visit is completed. The problem combines aspects of both the stochastic knapsack problem with uncertain item sizes and the OP. The stochastic orienteering problem was introduced in [71] where an $O(\log \log B)$-approximation algorithm was presented.
   - The Orienteering Problem with Stochastic Travel and Service Times (OPSTS) which was introduced in [28], wherein both travel and service times are stochastic. If a node is visited, a reward is received, but if it is not, a penalty may be incurred. This problem reflects the challenges of an employee of a company who, on a given day, may have more customers to visit than he can serve. In [28] heuristics for general problem instances and computational results for a variety of parameter settings were given.

4. The OP with Compulsory Vertices (OPCV) discussed in [66], models the variant of OP in which it is mandatory to visit a subset of the nodes. In TTDP modeling, these compulsory nodes may be significant POIs that should be included in any itinerary. Gendreau et al. ([66]) developed a branch-and-cut algorithm to solve to optimality problem instances with up to 100 nodes, some of which are compulsory.

5. The Orienteering Problem with Variable Profits (OPVP) introduced in [52], is a variant of the OP, in which the collection of profits at a node requires either a number of discrete passes or a continuous amount of time to be spent at the node. The problem can be used to model variants of TTDP where multiple visits or a longer stay at a location may be required to collect more profit. In [52] a unified branch-and-cut algorithm for the two versions of the OPVP is given, while computational experiments on instances adapted from TSPLIB show that the problem can be solved for a small number of nodes.

6. The Orienteering Problem with Hotel Selection (OPHS) is a new variant of OP introduced by Divsalar et al. [49]. The problem can be used to model the design of a multi-day tourist trip through an attractive region. The goal is to determine a fixed number of connected trips that visits some nodes and maximizes the sum of the collected profits (hotels are

not associated with any profit). Each trip is limited in length and should start and end in one of the hotels. The OPHS is approached using a skewed Variable Neighborhood Search (VNS) that consists of a constructive initialization procedure and an improvement procedure. The experimental results report a small average gap from optimal solutions, while tours are derived in time suitable for online applications. In [48] Divsalar et al. rename OPHS to Orienteering Problem with Intermediate Facilities (OPIF) and propose a memetic algorithm for solving it. Experimental results show that this algorithm compared to their VNS algorithm [49] gives better quality solutions and requires less computational effort especially for larger instances.

3 Multiple tour TTDP solution approaches

Archetti et al. ([10]) coined the term Vehicle Routing Problem with Profits (VRPP) for the extension of TSPP to multiple tours. In VRPP, a variant of the classical VRP [5], visiting the whole set of nodes is not compulsory; a profit is collected when visiting a node, while the collection of the profits is distributed over several vehicles with limited capacity. Known variants of the VRPP is the Team Orienteering Problem (TOP), i.e., the extension of the OP to multiple tours, the Prize-Collecting VRP (PCVRP), the Capacitated Profitable Tour Problem (CPTP) [8], and the VRP with profits and time deadlines (VRPP-TD). In PCVRP the main objective is a linear combination of three objectives: minimization of total distance traveled, minimization of vehicles used, and maximization of prizes collected [121]. In CPTP the objective is to maximize the difference between the total collected profit and the total travel cost [8]. In VRPP-TD, in addition to the capacity constraints, there are node-specific temporal constraints referred to as time deadlines. The objective function is the same with the function of CPTP [2].

Since TOP more closely formulates the multiple tour version of the TTDP than the other three VRPP variants mentioned above, in the sequel of this section we formally define TOP and we survey algorithmic approaches for solving the problem as well as extensions and variants of the problem (see Figure 5).

3.1 Team Orienteering Problem (TOP)

The extension of the OP to multiple tours was defined as the Team Orienteering Problem by Chao et al. [30]. The TOP first appeared in the literature with the name Multiple Tour Maximum Collection Problem (MTMCP) by Butt and Cavalier [26]. TOP is an extension of OP where the goal is to find k paths (or tours) each with length bounded by B, that have the maximum total collected profit (each non-starting, non-terminal node is visited at most once along the k paths) (see Figure 6). TOP is NP-hard and APX-hard since OP is a special case of TOP.

TOP can be formulated as an integer programming problem as follows [127]: Further to the notation for the OP, given the integer k, let x_{ijm} be equal to 1 if node i is followed by node j in path m or equal to 0 otherwise, y_{im} be equal to 1 if node i is visited in path m or equal to 0 otherwise and u_{im} be the position of node i in path m. With this notation we have the following formulations:

[5]VRP can be described as the problem of designing optimal delivery or collection routes from a depot to a number of nodes subject to certain constraints. The most common constraints are (i) capacity constraints, i.e., a demand is attached to each node and the sum of weights loaded on any route may not exceed the vehicle capacity, (ii) time constraints over individual routes, (iii) time windows, and (iv) precedence relations between pairs of nodes. Most variants of VRP assume that all nodes must be visited and there is no profit collected when visiting a node.

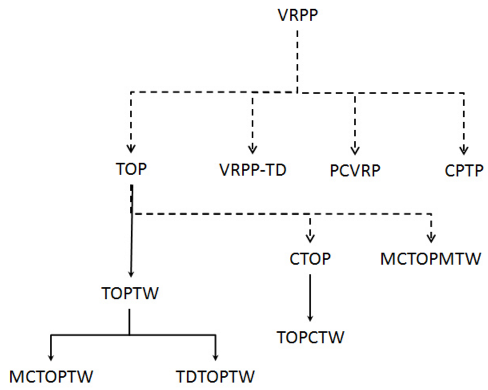

Figure 5: Multiple-tour TTDP solution approaches (solid arrows denote problem extensions, while dashed arrows denote variants).

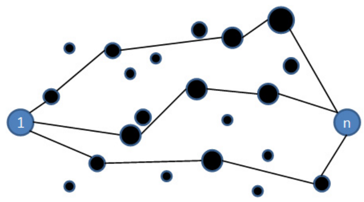

Figure 6: TOP illustration. Circles' radius denote nodes' profit.

$$\max \sum_{m=1}^{k} \sum_{i=2}^{N-1} p_i y_{im}, \tag{8}$$

s.t.

$$\sum_{m=1}^{k} \sum_{j=2}^{N} x_{1jm} = \sum_{m=1}^{k} \sum_{i=1}^{N-1} x_{iNm} = k, \tag{9}$$

$$\sum_{m=1}^{k} y_{rm} \leq 1, \text{ for all } r = 2, \ldots, N-1, \tag{10}$$

$$\sum_{i=1}^{N-1} x_{irm} = \sum_{j=2}^{N} x_{rjm} = y_{rm}, \text{ for all } r = 2, \ldots, N-1, m = 1, \ldots, k \tag{11}$$

$$\sum_{i=1}^{N-1} \sum_{j=2}^{N} c_{ij} x_{ijm} \leq B \text{ for all } m = 1, \ldots, k, \tag{12}$$

$$2 \leq u_{im} \leq N, \text{ for all } i = 1, 2, \ldots, N, \; m = 1, \ldots, k \tag{13}$$

$$u_{im} - u_{jm} + 1 \leq (N-1)(1 - x_{ijm}), \text{ for all } i, j = 2, \ldots, N, \; m = 1, \ldots, k \tag{14}$$

$$x_{ijm}, y_{im} \in \{0, 1\}, \text{ for all } i, j = 1, \ldots, N, \; m = 1, \ldots, k \tag{15}$$

The objective function (8) is to maximize the total profit of visited nodes. Constraints (9) and (10) ensure that each of the k paths starts at node 1 and ends at node N and that each non-starting, non-terminal node is visited at most once. Constraint (11) ensures that a flow of one unit can pass along each solution path connecting node 1 and node N, thereby ensuring that the path is connected. Constraint (12) ensures that the path meets the time budget. Finally, constraints (13) and (14) ensure that there are no closed subtours.

Exact algorithms for the TOP are presented by Butt et al. [27] and Boussier et al. [25]. Butt et al. [27] give an algorithm that optimally solves TOP by solving the relaxation of the problem with the column generation technique together with a branch and bound technique for deriving increasingly better solutions. Specifically, the problem is formulated as a set-partitioning problem and then a column generation procedure is applied. When applying the branch and bound technique, the solution space is partitioned around a specific node pair $\{u, v\}$ with one subspace containing solutions where both u, v belong to the same tour and the other one containing solutions where these two nodes cannot be part of the same tour. The combination of column generation and branch-and-bound (also known as branch-and-price in the literature) has also been applied in [25] for optimally solving the TOP. The selection of the new columns to be included at each step of column generation is reduced to solving an instance of the Elementary Shortest Path Problem with Resource Constraint by using a dynamic programming approach. Finally, in a branch and bound phase, different branches are created according to either whether a node should be visited or not or whether a particular edge should be included in a tour or not.

Blum et al. [23] present an approximation algorithm for variants of TOP in undirected graphs, where the paths have a common start point and not a fixed end point or they are mutually disjoint. Their main idea is to iteratively apply algorithms for rooted OP setting already visited node profits to zero. For the former case, applying this procedure using an α-approximation algorithm for rooted OP, an $1/(1 - e^{-\alpha})$ approximation ratio is obtained. In the latter case where the paths are mutually disjoint, using an α-approximation algorithm for rooted OP, an $(\alpha + 1)$ approximation ratio is obtained.

In the sequel, we outline the most important heuristic approaches for the TOP (see Table 3). The first heuristic algorithm (BC) for the TOP was presented by Butt and Cavalier [26]. They proposed a greedy algorithm that constructs the k tours successively. Every pair of nodes obtains a weight that gives an estimate of how advantageous it is to include both nodes in the same tour. Every tour initially contains the depot and the node pair with the greater weight. Then, at each step the node belonging to the heaviest pair of nodes with one of these nodes already in the tour is added to tour provided that this insertion is feasible.

The heuristic algorithm (CGW) for the TOP presented by Chao et al. in [30] extends the one presented by the same authors for the OP in [29]. There are two main differences between the two algorithms: Firstly, in TOP the current solution contains the k (instead of one) most profitable paths. Secondly, in TOP there are two perturbation moves instead of one that holds for the OP. The first move is identical for TOP and OP. In the second move of the TOP algorithm, a number of nodes with the lowest profit are removed from the paths of the solution.

In [120] a tabu search heuristic (TMH) for the TOP is proposed by Tang and Miller-Hooks, comprising three basic steps: initialization, solution improvement and evaluation. TMH is embedded within an adaptive memory procedure that alternates between small and large neighborhood stages during the solution improvement phase. Both random and greedy procedures for neighborhood solution generation are employed, and infeasible as well as feasible solutions are explored in the process. The heuristic has been compared against CGW heuristic.

Archetti et al. [10] presented three metaheuristics solving the TOP. After defining a number of local search moves that can be applied in the solution space of the problem at hand, they present a tabu search heuristic and two variable neighborhood search [72] heuristics (the fast variable neighborhood search - FVN and the slow variable neighborhood search - SVN) which iteratively apply local search moves for gradually improving the solution derived at each step.

Table 3: TOP heuristic algorithms

Reference	Algorithm	Technique
Butt and Cavalier [26]	BC	Greedy insertions of node pairs with the highest aggregate weight.
Chao et al. [30]	CGW	Local search procedure to improve a greedily derived initial solution; two perturbation moves applied thereafter to further improve the solution.
Tang and Miller-Hooks [120]	TMH	Tabu search comprising three steps: initialization, solution improvement and evaluation.
Archetti et al.[10]	SVN, FVN, TS	Three metaheuristics that initially apply local search moves and then a tabu search and two VNS heuristics, respectively.
Ke et al. [78]	ASe, ADC, ARC, ASi	Four ant colony optimization methods.
Vansteenwegen et al. [128]	GLS	Greedy creation of an initial solution; improvements sought by local search and then guided local search procedures.
Vansteenwegen et al. [132]	SVNS	Skewed VNS approach applying a combination of intensification and diversification procedures.
Souffriau et al. [113]	GRASP	A randomized procedure which builds an initial solution, followed by a local search method.
Souffriau et al. [114]	FPR, SPR	GRASP with a path relinking extension (to avoid the independence of the different iterations of GRASP).
Bouly et al. [24]	MA	Memetic Algorithm enhanced with local search techniques.
Muthuswamy et al. [99]	DPSO	A population of particles (i.e. feasible tours) initially generated, then using discrete particle swarm optimization to head for more profitable tours.
Dang et al. [44]	PSOiA	A set of particles initially randomly located, then improved by a recombination step and local search procedure.

The authors compare their algorithms with TMH and CGW and they show that each of the proposed heuristics improves the performance of TMH and CGW on average. They also show that FVN represents a fair compromise between solution quality and computational effort.

An Ant Colony Optimization-based heuristic algorithm (ACO) is proposed by Ke et al. [78] for the TOP. Specifically, an iterative procedure is followed wherein the ants generate k feasible tours by succesively inserting promising edges from previous iterations associated with relatively low cost and high profit in their endnodes. Four methods, i.e., the sequential (ASe), the deterministic-concurrent (ADC), the random-concurrent (ARC) and the simultaneous (ASi) methods, are proposed to construct candidate solutions in the framework of ACO. The authors compare these methods with several existing approaches. The results obtained by ASe are as good as the results obtained by Archetti et al. [10], however they are faster to obtain. Therefore, it appears that ASe is a very good compromise between solution quality and computational effort.

A guided local search [133] metaheuristic algorithm (GLS) for the TOP is presented by Vansteenwegen et al. [128]. A solution to the problem is initialized as in CGW [30] and a local search procedure is applied to improve it. Finally, guided local search is employed to ameliorate the effectiveness of the local search. In [132] Vansteenwegen et al. propose a Skewed Variable Neighbourhood Search (SVNS) framework for the TOP. The algorithms apply a combination of intensification and diversification procedures. The diversification procedures remove a chain of points in each path. The available budget spread over different paths within the current solution is gathered into a single path in the new solution. The intensification procedures try to increase the score or to decrease the travel time in a path. The SVNS algorithm clearly outperforms the GLS algorithm.

In [113] the authors employ the Greedy Randomised Adaptive Search Procedure (GRASP) to solve TOP. GRASP is a metaheuristic originally introduced by Feo and Resende [55]. GRASP performs a number of iterations that consist of a constructive procedure followed by a local search approach. The constructive procedure, based on a ratio between greediness and randomness, inserts nodes one by one until all paths are full. Thus, a new initial solution is generated during every iteration. Then, the initial solution is improved by the local search procedure which alternates between reducing the total time of the solution and increasing its total profit, until the solution is locally optimal. The different iterations are independent and the best solution found is saved and returned as a result. In [114] Souffriau et al. introduce a GRASP with Path Relinking metaheuristic approach for solving the TOP. The goal of the Path Relinking extension is to avoid the independence of the different iterations of the GRASP by adding a memory component, i.e. a pool of elite solutions consisting of a number of best solutions. At each iteration the best solution, considered for insertion into the pool of elite solutions, is returned by a procedure that takes as arguments a starting solution and a guiding solution and visits the solutions on the virtual path in the search space that connects the starting and the guiding solution. A fast variant (FPR) and a slow variant (SPR) of the approach are tested and compared against other state-of-the-art approaches. The quality of the results of the slow variant is comparable to the quality obtained by the best algorithms of Archetti et al. [10] and Ke et al. [78].

Bouly et al. [24] propose a memetic algorithm (MA) for the TOP enhanced with local search techniques. A population of chromosomes is constructed where a chromosome is a sequence of nodes from which a solution to TOP is obtained by applying a PERT[6] like technique. A child chromosome is produced by a couple of chromosomes by applying a crossover technique followed by a local search procedure with a certain probability. Computational results are compared with those of different methods such as CGW, TMH, the slow VNS algorithm (SVN), and the sequential method in the framework of ACO. It appears that MA outperforms SVN in terms of efficiency and is quite equivalent in terms of stability [24])

[6] Program Evaluation Review Technique

Muthuswamy et al. [99] tackle the TOP using discrete particle swarm optimization (DPSO), creating one tour at a time. At each step a population of particles is generated such that each particle represents a feasible tour. Then, using PSO particles are heading for more profitable solutions (tours). The whole procedure is enhanced with local search techniques. The quality of DPSO solutions have been compared against seven TOP heuristic algorithms (CGW [30], TMH [120], SVN [10], ACO [78], GLS [128], MA [24], FPR and SPR [114]). DPSO was found competitive and robust across all used benchmark problem sets.

In [44] a particle swarm optimization inspired algorithm (PSOiA) is presented for the TOP. A position of a particle is a permutation of the nodes of the graph from which a solution of the TOP is obtained using a split procedure. Initially, the set of particles is randomly located. Then, iteratively every particle's location is either replaced by a new one constructed by a randomized heuristic algorithm or is updated recombining its current and its local best position as well as the global best position found. At the end of each iteration a local search procedure is applied to each particle with a certain probability in order to improve the solution. The algorithm outperforms all the previously proposed heuristics for TOP, yielding a relative error of 0.0005% from the best known solution on the instances presented in [30].

In the survey article of Vansteenwegen et al. [127], a summary of the performance of the best TOP algorithms is given. The comparisons are based on 157 benchmark instances [30]. For each algorithm, the number of times the best known solution is found, is given together with the average gap to the best solution and the average computational time.

3.2 Team Orienteering Problem with Time Windows (TOPTW)

The TOP with Time Windows (TOPTW) introduced by Vansteenwegen P. [125], extends TOP adding the constraint of limited time availability of serviced nodes (this corresponds to the opening and closing hours of a POI). Exact solutions for the TOPTW are feasible for graphs with very restricted number of nodes (e.g. see the work by Z. Li and X. Hu [91] which is used on networks of up to 30 nodes).

Given the complexity of the problem, the main body of TOPTW literature exclusively involves heuristic algorithms. Notably, existing methods are metaheuristics that involve, (a) an insertion step (adds a visit to one of the k tours) iteratively performed until the solution (or a set of solutions) cannot be further improved, and (b) a diversification step that aims at escaping from local optima. Those two steps are repeated until a termination criterion is met. Depending on the insertion step principle, existing methods are designated either as deterministic (those that always produce the same solution for given problem instances) or as stochastic or probabilistic (those that involve a degree of randomness in solutions generation). Probabilistic methods are generally shown to yield high quality solutions (as they perform a more extensive search of the solution space) at the expense of increased execution time.

Labadi et al. [83] propose a local search heuristic algorithm for the TOPTW based on a variable neighbourhood structure. In the local search routine the algorithm tries to replace a segment of a path with nodes not included in a path that offer more profit. For that, an assignment problem related to the TOPTW is solved and based on that solution the algorithm decides which arcs to insert in the path.

Lin et al. [92] propose a heuristic algorithm based on simulated annealing (SA) for the TOPTW. On each iteration a neighbouring solution is obtained from the current solution by applying one of the moves swap, insertion or inversion, with equal probability. The new solution is adopted and becomes the current one, if it is more profitable than the currently best found solution, else it is accepted with some probability which is decreasing with increasing profit loss. After applying the above procedure for a certain number of iterations the best solution found so far is further improved by applying local search.

The Iterated Local Search (ILS) heuristic proposed by Vansteenwegen et al. [129] is the fastest known algorithm proposed for TOPTW [127]. ILS defines an "insertion" and a "shake" step. The insertion step adds, one by one, new nodes to a tour, ensuring that all subsequent nodes (those scheduled after the insertion place) remain feasible, i.e. they still satisfy their time window constraint. For each node i that can be inserted, the cheapest insertion time cost is determined. For each of these nodes, the heuristic calculates a ratio, which represents a measure of how profitable it is to visit i versus the time delay this node incurs. Among them, the heuristic selects the one with the highest ratio for insertion. The shake step is used to escape from local optima. During this step, one or more nodes are removed in each tour looking for non-included nodes that may either decrease the tour time length or increase the overall collected profit.

Montemanni and Gambardella proposed an ant colony system (ACS) algorithm [97] to derive solutions for a hierarchical generalization of TOPTW, wherein more than the k required routes are constructed. At the expense of the additional overhead, those additional fragments are used to perform exchanges/insertions so as to improve the quality of the k tours. The algorithm comprises two phases:

- Construction phase: Ants are sent out sequentially; when at node i, an ant chooses probabilistically the next node j to visit (i.e. to include into the tour) based on two factors:
  - The pheromone trail τ_{ij} (i.e. a measure on how good it has been in the past to include arc (i, j) in the solution).
  - The desirability n_{ij}, (a node j is more desirable when it is associated with high profit, it is not far from i, and its time window is used in a suitable way).
- Local search: performed upon the solutions derived from construction phase, aiming at taking them down to a local optimum.

Tricoire et al. [122] deal with the Multi-Period Orienteering Problem with Multiple Time Windows (MuPOPTW), a generalization of TOPTW, wherein each node may be assigned more than one time window on a given day, while time windows may differ on different days. Both mandatory and optional visits are considered. The motivation behind this modelling is to facilitate individual route planning of field workers and sales representatives. The authors developed two heuristic algorithms for the MuPOPTW: a deterministic constructive heuristic which provides a starting solution, and a stochastic local search algorithm, the Variable Neighbourhood Search (VNS), which considers random exchanges between chains of nodes.

Labadi et al. [84], [85] recently proposed a method that combines the greedy randomized adaptive search procedure (GRASP) with the evolutionary local search (ELS). GRASP generates independent solutions (using some randomized heuristic) further improved by a local search procedure. ELS generates multiple copies of a starting solution (instead of a single copy generated in ILS) using a random mutation (perturbation) and then applies a local search on each copy to yield an improved solution. GRASP-ELS derives solutions of comparable quality and requires significantly less computational effort to ACS. Compared to ILS, GRASP-ELS gives considerably better quality solutions at the expense of increased computational complexity.

Hu and Lim [73] propose an iterative three-component heuristic (I3CH) for TOPTW. To the best of our knowledge, I3CH yields the highest quality solutions among existing TOPTW solvers, although with prolonged execution time. The algorithm iteratively applies a local search procedure, a simulated annealing procedure and a route recombination step. Each route of a solution obtained from the local search and simulated annealing procedure is inserted into a pool of routes. Then, in the route recombination step, k disjoint routes from the pool with the highest total profit are picked, hence deriving a high quality solution.

Table 4 summarizes the performance of ILS, GRASP-ELS, ACS and I3CH compiled from the analytical results presented in [73]. This comparison is based on a number of benchmarks: sets c100, r100, rc100 and c200, r200, rc200 designed by Solomon [111] and pr01-10, pr11-20 designed by Cordeau et al. [40]. The table reports for each method and for different number of tours k ($k = 1, \ldots, 4$), the average gap to the best known solution and the average computational time, over all instance sets. Overall, ILS represents a fair compromise in terms of speed (less than 7 sec for $k = 4$ tours) versus deriving routes of reasonable quality (on average, less than 5% gap from the best known solution). ACS slightly improves the quality of derived solutions at the expense of prolonged execution time, practically prohibitive for online applications. GRASP-ELS derives solutions of comparable quality to ACS, but it needs more computational effort than ILS (especially for larger numbers of tours). I3CH yields the highest quality solutions; however, it requires a considerable amount of execution time (on average $\sim$ 250 sec), tens of times longer than ILS.

Table 4: Comparison of metaheuristic approaches on TOPTW

# of tours k	ILS		GRASP-ELS		ACS		I3CH	
	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)	Gap (%)	Time (s)
k=1	4.66	1.13	1.51	3.2	3.41	301	1.72	99.5
k=2	4.72	2.68	1.90	7.9	3.35	641.2	1.13	271.28
k=3	4.78	4.4	1.69	10.1	3.2	595	0.38	286.5
k=4	4.35	6.18	1.54	24.95	2.58	560.7	-0.13	373.7

Gavalas et al. recently presented CSCRatio and CSCRoutes, two cluster-based approaches to the TTDP [64]. The main incentive behind these approaches is to favor visits to topology areas featuring high density of good candidate nodes. Furthermore, they both favor solutions with reduced number of long transfers among nodes, which are associated with public transportation transfers in typical urban settings (such transfers are costly, time consuming and usually less attractive to tourists than short walking transfers). The comparison of CSCRatio over the ILS algorithm demonstrated that CSCRatio achieves higher quality solutions in comparable execution time (especially when considering limited itinerary time budget), while also reducing the average number of transfers. As regards the comparison of CSCRoutes over ILS, this confirmed the prevalence of the former in situations where the reduction of inter-cluster transfers is of critical importance. The lower number of transfers in CSCRoutes is achieved at the expense of slightly lower quality solutions. Furthermore, CSCRoutes achieves the best performance results with respect to execution time, compared to ILS and CSCRatio. Notably, the performance gap of the algorithms over ILS increases when tested on realistic TTDP instances, wherein nodes typically feature wide, overlapping time windows and are located nearby each other, while the daily time budget is 5-10h.

3.3 Time Dependent Team Orienteering Problem with Time Windows (TDTOPTW)

TDTOPTW is the problem that better models more complicated and realistic TTDP requirements among all problems and approaches surveyed in this article. TDTOPTW is particulary complex as it adds time dependency of arcs to TOPTW. Zenker et al. [138] described a tourism-inspired problem that refers to TDTOPTW and presented ROSE, a mobile application assisting pedestrians to locate events and locations, moving through public transport connections. ROSE incorporates three main services: recommendation, route generation and navigation. The authors identified the route planning problem to solve and they described it as a multiple-constrained destination recommendation with time windows using public transportation. However, no algorithmic solution to this problem has been proposed.

The work of Garcia et al. [60], [62] is the first to address algorithmically the TDTOPTW and is based on the algorithm by Vansteenwegen et al. [129] for the TOPTW. The authors present two different approaches to solve TDTOPTW, both applied on real urban test instances. The first approach involves a pre-calculation step, computing the average travel times between all pairs of POIs, allowing reducing the TDTOPTW to a regular TOPTW. A repair procedure introduces the real travel times between the POIS of the derived TOPTW solution. In case that the TOPTW solution is infeasible (due to violating the time windows of POIs included in the solution), a number of visits are removed. The second approach considers direct public transportations, without transfers, and assumes only periodic service schedules. It modifies the insert procedure of the TOPTW ILS heuristic [129] by introducing a few new concepts and formulas to keep the concepts updated, and making possible the local and efficient evaluation of the possible insertion of an extra POI. The authors propose two variants of the second approach that take transfers into account: (i) The fist variant is based on precalculating all travel times for each pair of POIs and for all required leave times. To reduce the number of calculations, the notion of the "period of a transfer connection" is used, defined as the least common multiple of the periods of all services involved in the transfer. (ii) The second variant models transfers as direct connections. The waiting time at the transfers is approximated by half of the period of the second service of the transfer.

The authors tested all approaches for a set of instances based on real data for a city with around 50 POIs and with high frequency of public transportation. Based on the results of the tests the following can be concluded:

- The second approach (real travel time with no transfers) gives good solutions only for cities with a small number of POI to POI connections that are unfeasible without transfers. The approach needs low computation time (the same order of magnitude with the TOPTW algorithm [129]).
- In the case that the average travel times are good approximations of the real travel times, the first approach (average travel time approach) gives only slightly worse solutions compared to the second approach and its variants (real travel time approaches). This happens only when we have high frequency of public transportation. The computation time of the first approach is comparable with the one of the second approach.
- Both variants of the second approach that take transfers into account, improve the results obtained by the second approach i.e., the real travel time approach with no transfers (considering transfers widens the search space and leads to better results [62]). The first variant, i.e. the real time approach based on the precalculation, is not appropriate for big cities with a large number of POIs, as a lot of memory is required to store the precalculated values and retrieving the values is too time consuming. The second variant is less accurate than the first one but it is more suitable for bigger cities.

Gavalas et al. [65] proposed two cluster-based heuristics (the Time Dependent CSCRoutes (TDCSCRoutes) and the SlackCSCRoutes) for solving the TDTOPTW which make no assumption on periodic service schedules. The main design objectives of the two algorithms are to derive high quality TDTOPTW solutions (maximizing tourist satisfaction), while minimizing the number of transit transfers and executing fast enough to support online web and mobile applications.

The prototyped algorithms have been tested in terms of various performance parameters (solutions quality, execution time, number of transit transfers, etc) upon real test instances compiled from the wider area of Athens, Greece. The performance of the algorithms has been compared against two variants that use precalculated average travel times (among the individual time dependent, real travel times) between POIs, the AvgILS and the AvgCSCRoutes. AvgILS refers to the average travel time approach proposed by Garcia et al. [62]. AvgCSCRoutes uses CSCRoutes [64] to construct routes based on pre-computed average travel times. With respect to the overall collected profit, TDCSCRoutes has been shown to perform marginally better. On the other hand, SlackCSCRoutes achieves a fair compromise among all the performance aspects. In practical applications, comprising very large datasets, AvgCSCRoutes could be the most suitable choice as it efficiently derives solutions of reasonably good quality. Nevertheless, its suitability largely depends on the high frequency of public transit services, so that average travel times represent a good guess.

3.4 Variants of TOP

In this subsection we present a number of TOP variants that have been introduced in the literature to model versions of TTDP. Those variants involve more compex formulations and take into account, for example, more attributes or multiple constraints on different attributes of the problem.

- The Capacitated Team Orienteering Problem (CTOP). Archetti et al. [8] introduced the CTOP as a TOP with an additional attribute, i.e., a nonnegative demand is associated with each node and an additional constraint, i.e., the total demand in each tour may not exceed the given capacity constraint. They present exact and heuristic algorithms that are extentions of schemes for solving the TOP. The exact algorithm is an adaptation of a branch-and-price scheme first presented in [25], while the heuristic algorithms are based on the heuristic solutions for the TOP given in [10]. In [5] a new branch-and-price scheme is presented to solve the CTOP. A column-based heuristic is applied at each node of the branch-and-bound tree in order to obtain primal bound values.

  Luo et al. [93] introduced a heuristic algorithm for the CTOP applying two different priority rules for inserting a node in a tour. By applying a local search procedure and according to the priority rule currently in use, the algorithm iteratively inserts the unvisited node with the highest priority in the tour. If the solution is not improved for a specified number of iterations the solution is perturbed and the priority rule is changed. The algorithm is compared against the exact and the heuristic algorithms proposed by Archetti et al. [8] on instances introduced in the same article. The results indicate that Luo et al's algorithm outperforms the other heuristics both in solutions' quality and execution time, and also finds the optimal solutions in the instances solved by the exact algorithm within an hour of execution time.

- The Team Orienteering Problem with Capacity Constraint and Time Window (TOPCTW). Li and Hu formulated the TOPCTW [91] which extends CTOP adding the constraint of limited time availability of serviced nodes, and obtained exact solutions using an integer linear programming solver. However, this approach is inappropriate for real-time applications.

- The Multiple agents maximum collection problem with time dependent rewards. Ekici and Retharekar [51] considered the problem as a variant of TOP, where each node's profit decreases linearly over time and the objective is to find k tours with the maximum difference between collected profit and travel cost. The authors formulate the problem as a mixed integer programming, and propose four auxiliary heuristic algorithms for creating a single tour as well as the main heuristic algorithm *Cluster-and-Route Algorithm*. Cluster-and-Route Algorithm first partitions the nodes into k sets using the k-means algorithm, then it constructs a tour within each cluster by applying one of the four auxiliary heuristics and finally improves the solution by either inserting new nodes in tours or rearranging the order of the existing nodes in a tour or transferring a node from one tour to another. Two sets of tests have been performed. Initially the four heuristic algorithms are compared against the solution obtained by the exact algorithm (Mixed Integer Programming based algorithm) allowed to run for four hours at most, on randomly generated single tour instances. The reward obtained by the heuristics was about 10% less than the reward obtained by the exact algorithm, while their execution time is negligible when compared with the exact algorithm. Then in a second set of tests, Cluster-and-Route Algorithm is compared with the exact algorithm allowed to run for four hours at most, on randomly generated instances and on modified instances of TOP [30]. In that scenario, the proposed heuristic has been found hundreds of times faster than the exact method, while yielding similar rewards for small instances, and greater for the large ones.

- The Multi-Constrained Team Orienteering Problem with Time Windows (MCTOPTW). Garcia et al. [60] introduced the MCTOPTW where each node is associated with a number of attributes; the sum of those attributes values is bounded by a max value (e.g. the sum of attractions entrance fee should not exceed an overall budget or the total time spent in parks cannot exceed a given time threshold). The proposed algorithm is based on ILS [129], incorporating two different aspects: (a) the feasibility check of visit insertions caters for checking constraints in addition to time feasibility; (b) the ratio function determining the candidate visit to be inserted is adapted so as to associate each attribute constraint with a special weight and include the available quantity of each constraint on the route. For instance, if the total entrance fee constraint is assigned a relatively high weight, the algorithm favors insertions of visits with relatively low entrance fee, even more so if currently selected visits sum to low overall fee (relatively to the fee threshold).

  Sylejmani et al. [119] proposed a tabu-search based algorithm for the MCTOPTW. Three steps are used to create neighbors: the Insert step which adds an unvisited node in a tour, the Replace step which replaces a node in a tour by an unvisited node and the Swap step which changes the positions of two nodes in the solution. In order to avoid revisiting neighboring solutions, the algorithm employs two tabu lists organized according to the recency and frequency of the steps above. The algorithm was compared against the algorithm of Souffriau et al [116] on instances introduced in [116], and was found to provide higher profit solutions within a few seconds on average.

- The Multi-Constraint Team Orienteering Problem with Multiple Time Windows (MCTOPMTW). Souffriau et al. [116] studied the MCTOPMTW, in effect an extension of MCTOPTW which allows defining different/ multiple time windows for different days. The proposed MCTOPMTW algorithm is based on a hybrid ILS-GRASP approach: GRASP yields an initial solution (GRASP involves a degree of randomness in the insertion phase) and the 'shake' routine of ILS is used thereafter to derive an improved solution. The authors report that the ILS-GRASP algorithm yields fairly quality solutions, while achieving computation time suitable for online applications.

4 Further algorithmic approaches for modeling TTDP variants

In Sections 2 and 3 we discussed TSPP and VRPP variants, respectively, which closely match the modeling requirements of the TTDP variants most frequently encountered in the literature. However, a number of other problems investigated in the optimization algorithms literature, could also capture various aspects and modeling parameters of TTDP variants and closely related problems. Admittedly, those problems deviate from a "strict" definition of the TTDP, however, they serve as viewpoint for a multifaceted investigation of the TTDP and offer intuitions for extending the functionality of existing TTDP solvers. Algorithmic approaches to solve such problems are reviewed herein, explaining their utility in addressing TTDP variants and closely related problem requirements.

The Distance constrained Vehicle Routing Problem (DVRP) and the Minimum Path Cover Problem (MPCP), both variants of the classical VRP can formulate useful variants of TTDP. In DVRP, given a depot node r and a distance constraint D the goal is to find a minimum cardinality set of tours originating from r and corresponding to routes for vehicles, that covers all the nodes in the network [86], [88], [101]. Each tour is required to have length at most D. DVRP may formulate the following problem: Given a set of POIs, we are asked to determine the minimum number of days that will be needed to visit all POIs without violating the constraint of the available time per day. The unrooted version of DVRP, defined as the MPCP in [12], seeks for the minimum number of paths each of length at most D, that cover all the nodes of the network. Note that in MPCP, the paths may start and end at any two nodes. MPCP can be reduced to DVRP by adding a depot node that is located at some large distance L from all nodes, and setting the distance constraint to $D + 2L$. DVRP was studied in [88] under the objectives of total distance and number of tours. It was shown that the optimal solutions under both objectives are closely related, and any approximation guarantee for one objective implies a guarantee with an additional loss factor of 2, for the other objective. In [101] the authors presented an $(O(\log 1/\epsilon), 1 + \epsilon)$-approximation algorithm: i.e., for any $\epsilon > 0$, the algorithm provides a solution violating the length bound by a $1 + \epsilon$ factor, while using at most $O(\log 1/\epsilon)$ times the optimal number of tours. The algorithm partitions the nodes of the network into subsets, according to their distance from the depot, and solves the unrooted DVRP with appropriate distance bounds on each subset. To solve the unrooted DVRP the $3-$approximation algorithm for the minimum path cover problem of Arkin et al. [12] is employed that proceeds as follows. First, it guesses the solution value of k and then finds k paths with total length at most $2kD$ that cover the nodes of the network. Finally, it cuts the paths into smaller paths with length less than or equal to D.

In all the problems cited up to now in this survey, the sites/customers are represented by the nodes of a network. Also, the network nodes are associated with profits and/or demands. There is a limited body of literature relevant to arc routing problems with profits i.e., problems in which the sites/customers are represented by the arcs of a network and the profits/demands are associated with the arcs. In this survey, the interest in arc routing problems with profits is motivated by some TTDP variants whose modeling requires profits to be associated with the links of the network since some links may be more beneficial to be traversed than others. As an example we may consider the derivation of personalized bicycle trips. Based on the biker's personal interests, starting and ending point and the available time budget, a personalized trip can be composed using arcs that better match the cyclist's profile. Additional applications of arc routing to TTDP could be the selection of paths of higher scenic value (among the many available between pairs of POIs) as well as the exclusion of paths including environmentally burdened road segments in favor of longer detours through pedestrian zones. In the sequel, we briefly review the literature on arc routing problems with profits focusing on the ones that may be used to model TTDP variants. (For an extensive survey on arc routing problems with profits the reader is refereed to the recent work of Archetti and Speranza [11].) We distinguish among the single tour and the multiple tour arc routing problems with profits.

- The Prize-collecting Rural Postman Problem (PRPP), a single tour arc routing problem with profits, is defined in [4]. In PRPP the arcs are associated with profits and costs, and the objective is to find a tour that maximizes the difference between the collected profit and the travel cost. Note that PRPP is the arc routing counterpart of the profitable tour problem (PTP). Problems related to post delivery and garbage collection can be modeled using PRPP. The problem has been studied from the algorithmic point of view in [3] and [53].
- The Arc Orienteering Problem (AOP) is a single tour arc routing problem with profits introduced by Souffriau et al. in [115]. In AOP the arcs are associated with profits and travel costs and the goal is to find a route from a node 0 to a node n with maximum profit and total travel cost not higher than a given value. Souffriau et al. use the AOP to solve the problem of planning cycle trips in the province of East Flanders. Their solution approach is based on a Greedy Randomized Adaptive Search Procedure (GRASP) while experimental results are based on instances generated from the East Flanders network.
- The Team Orienteering Arc Routing Problem (TOARP), the extension to the multiple route case of the AOP, is introduced by Archetti et al. in [6] where a branch-and-cut approach for solving small instances of the problem is developed. The authors in [7] propose a matheuristic approach. Experimental results show that the algorithm gives an average percentage error with respect to the optimal solution which is lower than 1%.
- In [9], the Undirected Capacitated Arc Routing Problem with Profits (UCARPP), a multiple tour arc routing problem with profits, is considered. UCARPP is the arc routing counterpart of the capacitated TOP (CTOP): a profit and a nonnegative demand is associated with each arc and the objective is to determine a path for each available vehicle in order to maximize the total collected profit, without violating the capacity and time limit constraints of each vehicle. Archetti et al. [9] consider an application where carriers can select potential customers for transporting their goods. Another potential application is the creation of personalized bicycle trips. An exact approach for solving the UCARPP and several heuristics were proposed in [9], while the problem was also studied by Zachariadis and Kiranoudis in [137] where a local search procedure was given.

The study of the combination of the orienteering problem and the arc routing problem with profits, under the name Mixed Orienteering Problem (MOP), where profits are associated to nodes as well as to arcs, is discussed in [127]. This problem is very interesting in the context of tourist trip planning as variants of MOP can be used to formulate TTDP variants where certain routes may be of tourist interest, in addition to attractions. To the best of our knowledge, the only relevant research works concern the one-period Bus Touring Problem (BTP) [46], and the Outdoor Activity Tour Suggestion Problem (OATSP) [94].

In BTP the objective is to maximize the total profit of the tour by selecting a subset of nodes to be visited and arcs to be traveled both having associated profits, given a constraint on the total touring time. The profit of recurrently visited nodes and arcs is only counted once. In [46] the BTP is reduced to the OP by means of a transformation of the original network. Unfortunately, the transformation process does not always guarantee a successful re-transformation from an OP solution to the corresponding BTP solution. In [46] two heuristic approaches are employed to solve the BTP: the Stochastic algorithm (S-algorithm) and the 1-step improvement algorithm. The S-algorithm proposed by Tsiligirides [123], uses Monte Carlo techniques to build a large set of routes based on a "desirability" factor of each node. From this set of routes the one with the maximum profit is selected as the OP solution. Then the 1-step improvement algorithm, a variant of an effective heuristic improvement process algorithm for the BTP proposed by Deitch and Ladany [45], is employed to improve the OP solution. The 1-step improvement algorithm inserts and deletes one node or one arc at a time (as opposed to the heuristic improvement process algorithm which builds candidate solutions from many nodes and/or arcs) depending on their "desirability" factor.

The OATSP, introduced recently by Maervoet et al. [94], involves finding attractive closed paths in a transportation network graph, tailored for a specific outdoor activity mode such as hiking and mountain biking. Total path attractiveness is evaluated as the sum of the average arc attractiveness and the profits of the nodes in the path. The problem involves finding a closed path of maximal attractiveness given a target path length and tolerance. That is, the OATSP requires a target path length instead of a maximal travel time required by the BTP. This gives rise to a path length window constraint. Maervoet et al. [94] present an efficient heuristic solution to the OATSP. Their method is based on spatial filtering, the evaluation of triangles in a simplified search space and shortest path calculation.

5 New prospects in tourist route planning problems

5.1 Arbitrary start/end tour locations

To the best of our knowledge all existing TTDP approaches, including relevant online web/mobile applications [39, 98], consider fixed start/end locations for tourist tours. Namely, the user is restricted to selecting the start/end locations of her daily tours among a pre-defined set of sites, typically coinciding with the user's accommodation, public transit hubs or selected tourist landmarks. However, this is not inline with the typical envisaged usage scenario, whereby the TTDP solver will be inquired by a mobile client; the tourist's starting location will be typically fixed to his current position and the ending location will be also defined arbitrarily by the user at query time. Clearly, the formulation of the (TD)TOP(TW) problem using precalculated travel costs among a fixed set of predefined locations/nodes (e.g. POIs and hotels) cannot support the above described dynamic usage scenario. Therefore, a generalization of the problem should be investigated, wherein the start/end locations of a tour may be any location in the destination city, i.e., they are both determined at runtime.

5.2 Modeling and solving more realistic TTDP formulations

The state-of-the-art relevant to the families of problems presented in previous sections, reveals that little has been done in regards with tourist trip design problems that have more complicated requirements and constraints, e.g. allowing modeling multiple user constraints and transfers through public transportation. This highlights a promising field of research which calls for modeling and solving extensions of TOPTW and TDTOPTW that take into account realistic TTDP issues or constraints like the following:

- Weather conditions: museums may be more appropriate to visit than open-air sites in rainy or relatively cold days, while the contrary may be true in sunny days; hence, route planning could take into account weather forecast information in recommending daily itineraries.

- Accessibility features of sites should be taken into account when recommending visits to individuals with motor disabilities.
- Tourists are commonly under inflexible budget restrictions when considering accommodation, meals, means of transport or visits to POIs with entrance fees. Hence, next to the time budget, money budget further constrains the selection of POI visits.
- Recommended tourist routes that exclusively comprise POI visits and last longer than a few hours are unlikely to be followed closely. Tourists typically enjoy relaxing and breaks as much as they enjoy visits to POIs. A realistic route should therefore provide for breaks either for resting (e.g. at a nearby park) or for a coffee and meal. Coffee and meal breaks are typically specific in number, while respective recommendations may be subject to strict time window (e.g. meal should be scheduled around noon) and budget constraints. A relevant approach which involves "virtual nodes" as candidate lunch locations is proposed in [130].
- The assumption of POIs having periodic time windows is invalid. POIs typically operate at specific days weekly, possibly with varying opening and closing hours. Hence, TTDP modelling should take into account multiple time windows.
- Max-n Type [112] constrains the selection of POIs by allowing stating a maximum number of certain types of POIs, per day or for the whole trip. e.g. maximum two museum visits on the first day. Likewise, mandatory visits (i.e. tours including at least one visit to a POI of certain type, such as a visit to a church) could also be asked for.

Modeling and solving of problems relevant to TTDP represents another promising research direction. For instance, hotel selection is often a cumbersome task for tourists unfamiliar with hotels and POI locations or with the structure of the public transportation network in the tourist destination area. This is even more true when planning long road trips across large geographic areas (in such scenarios, changing accommodation in daily basis is common) [126]. Several criteria could apply in hotel recommendation, including cost, amenities or cost-for-profit (i.e. select an affordable hotel suitably located so as to maximize the overall profit collected from POI visits throughout the whole trip). Restaurants selection is equally important as meal/dinner breaks are mandatory, while constrained by several - often contradictory - user preferences (e.g. budget, diet preferences, favorite cuisine) and restaurant characteristics (e.g. menu, price list, opening hours).

Another example is the problem of determining the minimum number of days that one needs to visit all selected POIs without violating the constraint of the available time per day. This problem may be formulated using the distance constrained vehicle problem (DVRP) described in Section 4. Other interesting variants of TTDP may be formulated using the mixed orienteering problem, also discussed in Section 4.

5.3 Fast tourist routes updates

Existing TTDP solutions deal with tourist queries for multiple days' route planning, considering routes with the same starting/ending location. However, there is no provision for user deviations from the originally planned routes, although such deviations are highly probable to occur.

Dynamic rescheduling functionality should detect route invalidation (infeasibility) and present a new route schedule in real time. This should exclude POIs already visited and recommend a tour for the remainder of the current day (starting from the user's current position) as well as the next days of stay at the destination.

5.4 Parallel computation

One of the most important objectives in the design of algorithmic methods for the TTDP is the real time response to user queries. Parallel computing is a promising approach for attaining this important objective. Considering all the solution methods for TTDP, heuristics and metaheuristics are most amenable to parallel computation since the huge solution space arising in this kind of problems enables a lot of variation in parallelizing solution searching. Specifically, according to [43] one could parallelize the local search for good neighboring solutions or partition the solution space in number of subspaces and run a heuristic in each of these subspaces, in parallel. Alternatively, a number of search threads could be created working on the same solution space, starting from different or the same initial solution and applying the same or different heuristics. These threads could work independently or could cooperate periodically exchanging information about their progress and the good solutions they have found so far. An interesting aspect of these approaches is that they may as well provide new heuristic solutions with improved solution quality since they can search the solution space and combine solutions in such a way that it is very costly to simulate with a sequential implementation. Although, parallel heuristics has been proposed in the literature for the VRP and TSP [35], [41], [42], [102], [118] parallel solutions for TTDP are missing and the design of new parallel heuristics for TTDP may solve the problem of the fast derivation of the tourist itineraries.

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