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Decision Support

Uncertain multiobjective traveling salesman problem

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ABSTRACT

Traveling salesman problem is a fundamental combinatorial optimization model studied in the operations research community for nearly half a century, yet there is surprisingly little literature that addresses uncertainty and multiple objectives in it. A novel TSP variation, called uncertain multiobjective TSP (UMTSP) with uncertain variables on the arc, is proposed in this paper on the basis of uncertainty theory, and a new solution approach named uncertain approach is applied to obtain Pareto efficient route in UMTSP. Considering the uncertain and combinatorial nature of UMTSP, a new ABC algorithm inserted with reverse operator, crossover operator and mutation operator is designed to this problem, which outperforms other algorithms through the performance comparison on three benchmark TSPs. Finally, a new benchmark UMTSP case study is presented to illustrate the construction and solution of UMTSP, which shows that the optimal route in deterministic TSP can be a poor route in UMTSP.

1. Introduction

Traveling salesman problem (TSP) is a well-known NP-hard problem in combinatorial optimization, almost every new approach for solving engineering and optimization problems has been validated on TSP. The first efficient algorithm for relatively big problems was presented just in the paper (Dantzig, Fulkerson, & Johnson, 1954). Since then, many theories and methods have been developed for solving TSP, such as Erdoğan, Cordeau, and Laporte (2010), Rego, Gamboa, Glover, and Osterman (2011) and Çela, Deineko, and Woeginger (2012). These traditional TSP studies mentioned above are all assumed in deterministic environment. However, in the real world, TSP situations are often indeterministic, some or all of the TSP's parameters are not known with certainty at the moment we have to make decision. With the great improvement of probability theory, the stochastic model has been widely used in many relevant TSPs to represent the indeterminacy, including the consideration of probability in the presence of customers (Jaillet, 1988), the demand level (Bertsimas and Simchi-Levi, 1996), the travel time (Kao, 1978), and the service time at customers' site (Chang, Wan, & Ooi, 2009), usually assuming a known distribution governs some of the problem's parameters.

Unfortunately, when the sample size in TSP is too small to estimate a probability distribution, the frequently used probability distribution is not always appropriate; especially when the information is vague, we have to invite some domain experts to evaluate their belief degree that each event will occur in this case. Take the unmanned aerial vehicle (UAV) reconnaissance mission planning for example, before we assign the UAV to fly over dangerous targets for reconnaissance missions, the flight time, fuel usage and threat from enemies on the flight path between targets are indeterministic, we can only obtain the belief degree of these quantities, rather than the probability which is on the basis of large sample size. Such types of indeterminacy are called uncertainty, which is ubiquitous in real-life situation, such as the transportation planning in disaster response, etc. A lot of surveys showed that human beings usually overweight unlikely events, and the personal belief degree may have much larger variance than the real frequency (Liu, 2012b). Liu (2012a) declared that it is inappropriate to apply both probability theory and fuzzy set theory to uncertainty, because both theories may lead to counterintuitive results in this case. In order to deal with such kind of uncertain problem, Liu (2009b) founded the uncertainty theory, which is a branch of mathematics based on normality, monotonicity, self-duality, and countable subadditivity axioms, as a mean of handling uncertainty that is due to imprecision rather than to randomness. For modeling indeterminacy, as pointed out by Liu (2011), there exist two

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mathematical systems, one is probability theory and the other is uncertainty theory. If the parameters involved in TSP are uncertain variables, the resulting problem is called an uncertain TSP (UTSP), and can be solved through uncertainty theory.

As we all know, the multiobjective optimization is one of most important optimization problems in real world, thus, the UTSP with multiple and conflicting objectives is much closer to real situation. For instance, considering the timeliness of information, successful accomplishment of a mission and the mission cost, the UAV reconnaissance mission consists in allocating a UAV to a set of predetermined locations or targets in the shortest time while minimizing the threat imposed by the enemies and the fuel usage during the mission. In order to reduce the mission risk, the UAV need to avoid the threat from enemy, while the distance between two targets may be extended and the mission time and fuel usage are increased further. Hence, a new type of TSP called uncertain multiobjective TSP (UMTSP) with multiple objectives under uncertain environment is proposed in this paper based on uncertainty theory and multiobjective optimization technique. Dealing with multiobjective programming problem, one of the most used approaches is to aggregate all the objectives in a function and convert the original problem into a single objective optimization problem, which needs to guarantee that the solutions obtained are Pareto efficient to the original problem. Similarly, a new solution approach named *uncertain approach* is put forward to generate Pareto efficient route of UMTSP in this paper, by transforming the UMTSP into a single objective UTSP first, and then into its equivalent deterministic problem, which will guarantee the converted single objective UTSP is still provided with the uncertain nature of original UMTSP. In order to prove the validity that the routes obtained by this new approach are Pareto efficient to the original UMTSP, the Pareto efficient route in UMTSP should be defined first.

Personally speaking, in order to assure the uncertain nature of UMTSP, the Pareto efficient route in UMTSP should be defined on the uncertain objectives directly. The symbol $\prec$ or $\preceq$ is used to denote the relationship between uncertain objectives. For instance, $f(\bar{\pi}(X),\xi)\preceq$ (or $\prec$)$f(\pi^*(X),\xi)$ means that the valuation of uncertain objective $f(\bar{\pi}(X),\xi)$ is lower than or equal (or strictly lower than) to that of uncertain objective $f(\pi^*(X),\xi)$, where the valuation is a function defined under certain principles that determine the value of uncertain objectives in UMTSP. As different real-life problems call for different meanings of uncertain objectives valuation to satisfy its need in practical application, corresponding principle $\mathcal{P}$ should be presented to define this valuation according to the real context of problem. For example, considering the UMTSP as a road system, since we want to minimize the travel time and cost in the long run, we would like to use the expected value of travel time and cost to define this valuation. In one paper we submitted recently (Wang, Guo, Zheng, & Yang, 2015), several consistent comparison methods are presented to define the relationship between uncertain variables, which use expected-value, α-optimistic value and α-pessimistic value of uncertain variables respectively. As the expected value of uncertain variable is widely used in real-life problem, in this paper expected-value principle $\mathcal{P}_{\mathbb{E}}$ is adopted to obtain $\mathcal{P}_{\mathbb{E}}$ Pareto efficient routes in UMTSP. Based on the $\mathcal{P}_{\mathbb{E}}$ principle and the definition of $\mathcal{P}_{\mathbb{E}}$ Pareto efficient route in UMTSP, it is proved that the optimal route obtained using *uncertain approach* is $\mathcal{P}_{\mathbb{E}}$ Pareto efficient route in original UMTSP, which shows that the optimal route obtained in deterministic TSP can be a very poor route in UMTSP.

Considering the uncertain and NP-hard nature in UMTSP, metaheuristics and evolutionary algorithms should be widely applied to UMTSP for successful generation of optimal routes. As a relatively new member of swarm intelligence, the artificial bee colony (ABC) algorithm is a meta-heuristic bionic algorithm based on the intelligent foraging behavior of honey bees proposed by Karaboga (2005). So far, due to its simplicity and ease of implementation, the ABC algorithm has been adopted by researchers in a variety of fields, including redundancy allocation problem (Guo, Wang, Zheng, & Wang, 2015), vehicle routing problem (Szeto, Wu, & Ho, 2011), etc., and that its effectiveness and efficiency on algorithm performance are competitive to other optimization algorithms has been experimentally validated (Karaboga & Basturk, 2008). In this paper, given the combinatorial character of UMTSP, a new ABC algorithm is designed for obtaining efficient routes in UMTSP, which uses reverse operator, crossover operator and mutation operator to improve the exploitation and exploration ability in basic ABC algorithm. Given the analogies between the TSP and the UMTSP, it is reasonable to expect good performance of the new ABC algorithm on the UMTSP if the proposed algorithm performs well on the TSP. In order to test the performance of the proposed ABC algorithm in TSP, some benchmark symmetric TSPs from TSPLIB (2012) are utilized for experiment, including *danzig42.tsp*, *st70.tsp* and *gr96.tsp*. Results show that it is applicable to solve UMTSP using the proposed ABC algorithm with the combination of *uncertain approach*.

The paper is organized in the following manner. In Section 2, some useful definitions and properties about uncertain theory with application to UMTSP are introduced. In Section 3, the UMTSP is described, and its corresponding mathematical model is presented. In Section 4, a new approach to generate $\mathcal{P}_{\mathbb{E}}$ Pareto efficient route of UMTSP based on the $\mathcal{P}_{\mathbb{E}}$ principle is proposed and its validity has been proved. In Section 5, after a brief introduction of the classic ABC algorithm, the new ABC algorithm for solving the UMTSP is introduced. In Section 6, an application case study is provided. Finally, the major results of this research are stated in Section 7.

2. Preliminaries

Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element Λ in $\mathcal{L}$ is called an event. A set function $\mathcal{M}$ from $\mathcal{L}$ to [0, 1] is called an uncertain measure if it satisfies the following axioms (Liu, 2007):

Axiom 1. (*Normality Axiom*) $\mathcal{M}\{\Gamma\}=1$ for the universal set Γ.

Axiom 2. (*Duality Axiom*) $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ for any event Λ.

Axiom 3. (*Subadditivity Axiom*) For every countable sequence of events $\Lambda_1,\Lambda_2,\ldots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\}\le\sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$$

Liu (2009a) proposed the fourth axiom of uncertainty theory called product measure axiom.

Axiom 4. (*Product Axiom*) Let $(\Gamma_k,\mathcal{L}_k,\mathcal{M}_k)$ be uncertainty spaces for $k=1,2,\ldots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty}\Lambda_k\right\}=\bigwedge_{k=1}^{\infty}\mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k=1,2,\ldots$, respectively.

The triplet $(\Gamma,\mathcal{L},\mathcal{M})$ is referred to as a uncertainty space (Liu, 2007), in which an uncertain variable is defined as follows:

Definition 2.1 (Liu, 2007)**.** An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma,\mathcal{L},\mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set

$$\{\xi\in B\}=\{\gamma\in\Gamma\mid\xi(\gamma)\in B\}$$

is an event.

Definition 2.2 (Liu, 2009a)**.** The uncertain variables $\xi_1,\xi_1,\ldots,\xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}(\xi_i\in B_i)\right\}=\bigwedge_{i=1}^{n}\mathcal{M}\{\xi_i\in B_i\}$$

for any Borel sets $B_1,B_2,\ldots,B_n$ of real numbers.

Definition 2.3 (Liu, 2009b). The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}$$

for any real number x.

Definition 2.4 (Liu, 2011). Let ξ be an uncertain variable with regular uncertainty distribution Φ. Then the inverse function Φ^{-1} is called the inverse uncertainty distribution of ξ.

Definition 2.5 (Liu, 2007). Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{\infty} \mathcal{M}\{\xi \ge x\}dr - \int_{-\infty}^{0} \mathcal{M}\{\xi \le x\}dr$$

provided that at least one of the two integrals is finite.

Definition 2.6 (Liu, 2012a). An uncertain variable ξ is called linear if it has a linear uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \le a \\ (x-a)/(b-a), & \text{if } a \le x \le b \\ 1, & \text{if } x \ge b \end{cases}$$

denoted by $\mathcal{L}(a, b)$ where a and b are real numbers with $a < b$.

Definition 2.7 (Liu, 2012a). An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \le a \\ (x-a)/2(b-a), & \text{if } a \le x \le b \\ (x+c-2b)/(2(c-b)), & \text{if } b \le x \le c \\ 1, & \text{if } x \ge c \end{cases}$$

denoted by $\mathcal{Z}(a, b, c)$ where a, b, c are real numbers with $a < b < c$.

Theorem 2.1 (Liu, 2009b). *Let $\xi_1, \xi_2, \ldots, \xi_n$ be uncertain variables, and f a real-valued measurable function. Then $f(\xi_1, \xi_2, \ldots, \xi_n)$ is an uncertain variable.*

Theorem 2.2 (Liu, 2011). *Let ξ be an uncertain variable with regular uncertainty distribution Φ. If the expected value exists, then*

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)d\alpha$$

Theorem 2.3 (Liu, 2011). *Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If the function $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is an uncertain variable with inverse uncertainty distribution*

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \Phi_{m+2}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)).$$

3. Description and mathematical formulation of UMTSP

In this section, a formal mathematical description of UMTSP is presented, where all input parameters are assumed to be uncertain variables. Firstly, some background on TSP that will be used further in the modeling of our UMTSP is provided.

3.1. TSP description

TSP is one of the most widely studied NP-hard combinatorial optimization problems, whose single objective version is a typical benchmark for metaheuristics. The TSP in its modern form assumes that a traveling salesman has to visit a number of given cities, starting and ending at the same city. Then we need to find the smallest route whereby a person could travel to each city precisely once and then return to his home city. In order to model TSP, it is represented by complete edge-weighted graph $G = (V, E)$, with V being set of $n = |V|$ nodes or vertices representing cities and $E \subseteq V \times V$ being set of directed edges or arcs. Each arc $(i, j) \in E$ is assigned the value of length d_{ij}, which is the distance between cities i and j with $i, j \in V$. In this paper the symmetric TSP is considered, where $d_{ij} = d_{ji}$ holds for all arcs in E. The goal in TSP is thus to find minimum-length Hamiltonian Circuit of graph, where Hamiltonian Circuit is a closed path visiting each of n nodes in G exactly once. Thus, an optimal route to TSP is the permutation $\pi(X)$ of node indices $X = 1, 2, \ldots, n$ such that length $f(\pi(X))$ is minimal, where $\pi(X) = \{K_1, K_2, \ldots, K_n\}$, and $f(\pi(X))$ is given by

$$f(\pi(X)) = \sum_{i=1}^{n-1} d(K_i, K_{i+1}) + d(K_n, K_1)$$

3.2. Formulation of UMTSP

Based on the TSP description introduced above, the formulation of UMTSP can be presented as follows:

$$\min_{\pi(X)} f(\pi(X), \xi) = (f_1(\pi(X), \xi^{(1)}), f_2(\pi(X), \xi^{(2)}), \ldots, f_p(\pi(X), \xi^{(p)})) \tag{3.1}$$

where $\pi(X) = \{K_1, K_2, \ldots, K_n\}$ is the permutation of node indices $X = 1, 2, \ldots, n$; $\xi^{(k)}$ is a $n \times n$ matrix with independent uncertain variables $\xi^{(k)}_{(i,j)}$, and $\xi^{(k)}_{(i,j)} = \xi^{(k)}_{(j,i)}$, $(k = 1, 2, \ldots, p; i = 1, 2, \ldots, n; j = 1, 2, \ldots, n)$; all the components of $\xi^{(1)}, \xi^{(2)}, \ldots, \xi^{(p)}$ are independent uncertain variables defined on the uncertain space $(\Gamma, \mathcal{L}, \mathcal{M})$.

As shown in model (3.1), the objective function in the UMTSP becomes dependent not only on the route solution $\pi(X)$, but also on an uncertain influence from uncertain variables.

Remark 1. In the p-objective UMTSP proposed above, p different uncertain cost factors $\xi^{(1)}, \xi^{(2)}, \ldots, \xi^{(p)}$ are defined between each pair of cities. In practical applications the uncertain cost factors may represent travel cost, distance, time, risk or tourist attractiveness, etc. Since the uncertain cost factors in the objective functions are completely independent, all objectives in the UMTSP we proposed are independent and non-correlated.

Remark 2. In the UMTSP model proposed above, the form of each objective function depends on the real application context we consider. For example, when the first objective is considered as the minimization of travel time, then $f_1(\pi(X), \xi^{(1)})$ can be formulated in a sum form as follows:

$$\min_{\pi(X)} f_1(\pi(X), \xi^{(1)}) = \sum_{i=1}^{n-1} \xi^{(1)}_{(K_i, K_{i+1})} + \xi^{(1)}_{(K_n, K_1)}$$

where $\xi^{(1)}_{i,j)}$ means the travel time between city i and j.

When the first objective is considered as the minimization of risk, then $f_1(\pi(X), \xi^{(1)})$ can be formulated in a product form as follows:

$$\min_{\pi(X)} f_1(\pi(X), \xi^{(1)}) = 1 - \prod_{i=1}^{n-1} \left(1 - \xi^{(1)}_{(K_i, K_{i+1})}\right) \cdot \left(1 - \xi^{(1)}_{(K_n, K_1)}\right)$$

where $\xi^{(1)}_{i,j)}$ means the risk between city i and j.

Obviously, there exists many other forms to formulate the objective functions in UMTSP, usually it is formulated according to the practical application considered.

Since the objectives usually conflict with each other in UMTSP, there is no optimal route that simultaneously minimizes all the objective functions. In this case, the concept of Pareto efficient route should be introduced in UMTSP, which means that it is impossible to improve any one of objectives without sacrificing on one or more of the other objectives. Thus a new solution approach should be proposed to obtain the Pareto efficient routes in UMTSP, named *uncertain approach*.

4. Uncertain approach

4.1. Definitions of relationship between uncertain variables

In order to define Pareto efficient routes in UMTSP on the uncertain objectives directly, the definitions of relationship between uncertain variables are presented.

Definition 4.1 (Wang et al. 2015)**.** Let ξ and η be two uncertain variables, we say $\xi \prec$ (*or* $\preceq$) η if and only if $\mathcal{P}[\xi] <$ (*or* $\leq$)$\mathcal{P}[\eta]$, where $\xi \preceq \eta$ (or $\xi \prec \eta$) means that the valuation of ξ is lower than or equal to (or strictly lower than) that of η, and $\mathcal{P}$ denotes the principle used to define the valuation of uncertain variable.

Remark 3. It is worth pointing out that for the comparison between uncertain variables, the relationship is defined under valuation of uncertain variables. Different real-life problems call for different meanings of valuation to satisfy its need in practical application, therefore, corresponding principle $\mathcal{P}$ should be proposed to define this valuation according to the real context of problem, where $\mathcal{P}$ is the generalization of meanings of this valuation. For example, in the UMTSP, since we want to minimize the travel time $T(\pi(X), \xi)$ in the long run, it is rational to use its expected value $E[T(\pi(X), \xi)]$ to define the relationship. If $T(\bar{\pi}(X), \xi) \prec T(\pi^*(X), \xi)$, then it means that $E[T(\bar{\pi}(X), \xi)] < E[T(\pi^*(X), \xi)]$. We call it the expected-value principle $\mathcal{P}_{\mathrm{E}}$, which will be defined in Definition 4.2.

Definition 4.2 (Wang et al. 2015)**.** Let ξ and η be two uncertain variables, we say $\xi \prec$ (*or* $\preceq$) η if and only if $E[\xi] <$ (*or* $\leq$)$E[\eta]$, where $E[\cdot]$ denotes the expected value of uncertain variable.

When defining the relationship between uncertain variables using Definition 4.2, we call it the expected-value principle $\mathcal{P}_{\mathrm{E}}$. Other definitions of relationship between uncertain variables and corresponding principles can be referred to the literature (Wang et al., 2015).

Remark 4. What kind of valuation principle $\mathcal{P}$ defines in the relationship between uncertain variables, then what kind of principle we call Definition 4.1. For example, when $\mathcal{P}$ defines the expected-value of uncertain variables, we call Definition 4.1 the $\mathcal{P}_{\mathrm{E}}$ principle, and solve UMTSP under $\mathcal{P}_{\mathrm{E}}$ principle. Obviously, the principle $\mathcal{P}$ used to define the valuation of uncertain objectives is of a great generality, it not only include the existing principles, but also those principles we have not constructed yet. Of course, the principle $\mathcal{P}$ cannot be constructed at will, usually it is constructed according to the need of practical application or the context of specific problems.

As the expected value of uncertain variable is widely used in real-life problem, the UMTSP is solved under $\mathcal{P}_{\mathrm{E}}$ principle to obtain $\mathcal{P}_{\mathrm{E}}$ Pareto efficient routes in this paper. For solving UMTSP under $\mathcal{P}_{\mathrm{E}}$ principle efficiently, four basic theorems are presented as follows:

Theorem 4.1 (Wang et al. 2015)**.** *Let ξ and η be two uncertain variables with regular uncertainty distributions Φ and Ψ respectively, if $\xi \prec$ (or $\preceq$) η, then for any real number $\lambda > 0$, we have $\lambda\xi \prec$ (or $\preceq$) $\lambda\eta$.*

Theorem 4.2 (Wang et al. 2015)**.** *Assume that ξ_1 and ξ_2 are two independent uncertain variables with regular uncertainty distributions Φ_1 and Φ_2, η_1 and η_2 are two independent uncertain variables with regular uncertainty distributions Ψ_1 and Ψ_2, if $\xi_1 \prec \eta_1$, $\xi_2 \preceq \eta_2$, then we have $\xi_1 + \xi_2 \prec \eta_1 + \eta_2$.*

Theorem 4.3 (Wang et al. 2015)**.** *Let ξ and η be two uncertain variables with regular uncertainty distributions Φ and Ψ respectively, if $\xi \prec$ (or $\preceq$) η, and the lower bounds of ξ and η, ξ_0 and η_0 exist, then for $t_0 = \min(\xi_0, \eta_0)$, we have $(\xi - t_0)^2 \prec$ (or $\preceq$) $(\eta - t_0)^2$.*

Theorem 4.4 (Wang et al. 2015)**.** *Let ξ and η be two uncertain variables with regular uncertainty distributions Φ and Ψ respectively, if $\xi \prec$ (or $\preceq$) η, and $\sqrt{\xi}$ and $\sqrt{\eta}$ exist, then we have $\sqrt{\xi} \prec$ (or $\preceq$) $\sqrt{\eta}$.*

4.2. Uncertain approach for UMTSP

In order to solve model (3.1), the Pareto efficient route in UMTSP should be defined first.

Definition 4.3. A feasible route $\pi^*(X)$ is said to be $\mathcal{P}_{\mathrm{E}}$ Pareto efficient to the UMTSP if there is no feasible route $\pi(X)$ such that

$$f_k(\pi(X), \xi^{(k)}) \preceq f_k(\pi^*(X), \xi^{(k)}), \quad k = 1, 2, \ldots, p$$

and $f_k(\pi(X), \xi^{(k)}) \prec f_k(\pi^*(X), \xi^{(k)})$ for at least one index k.

In order to obtain $\mathcal{P}_{\mathrm{E}}$ Pareto efficient route in model (3.1) and guarantee the uncertain nature of model (3.1), the model (3.1) is converted into a single objective UTSP using a real-valued measurable function F, that is

$$\min_{\pi(X)} U(\pi(X), \xi) = F(f_1(\pi(X), \xi^{(1)}), f_2(\pi(X), \xi^{(2)}), \ldots, f_p(\pi(X), \xi^{(p)})) \tag{4.1}$$

Note that since $U(\pi(X), \xi)$ is still an uncertain variable, the uncertain nature of UMTSP is guaranteed using *uncertain approach*. Before solving the model (4.1), we need to define the optimal route of it.

Definition 4.4. A feasible route $\pi^*(X)$ is called an optimal route to the model (4.1) if

$$U(\pi^*(X), \xi) \preceq U(\pi(X), \xi)$$

for any feasible route $\pi(X)$.

Obviously, the optimal route to model (4.1) is also defined under the relationship between uncertain variables.

Under $\mathcal{P}_{\mathrm{E}}$ principle the equivalent deterministic single objective TSP can be obtained as follows

$$\min_{\pi(X)} E[U(\pi(X), \xi)] = E[F(f_1(\pi(X), \xi^{(1)}), f_2(\pi(X), \xi^{(2)}), \ldots, f_p(\pi(X), \xi^{(p)}))] \tag{4.2}$$

According to Definition 4.4 and the requirement of Pareto efficiency, a road map can be drawn for the solution of UMTSP, that is, find a real-valued measurable function F to transform the UMTSP into a single objective UTSP first, then obtain its equivalent deterministic one under $\mathcal{P}_{\mathrm{E}}$ principle, and finally prove that the optimal solution to single objective UTSP (4.1) is still Pareto efficient to the original UMTSP (3.1).

Next, we will try to find the function F, and prove the validity of *uncertain approach* with application to UMTSP under $\mathcal{P}_{\mathrm{E}}$ principle. According to those well-known methods used for transforming the deterministic multiobjective programming problem into a deterministic single objective programming problem, the ideal point method is considered here.

4.3. Ideal point method

The method we proposed to convert the UMTSP (3.1) into a single objective UTSP (4.1) is the ideal point method, which is shown as follows:

$$\min_{\pi(X)} U(\pi(X), \xi) = \sqrt{\sum_{j=1}^{p} \left(f_j(\pi(X), \xi^{(j)}) - f_j^0\right)^2} \tag{4.3}$$

where f_j^0 denotes the lower bound of single objective $f_j(\pi(X), \xi^{(j)})$ $(j = 1, 2, \ldots, p)$ on feasible set without considering other objectives.

Theorem 4.5. *The optimal route to single objective UTSP (4.3) under $\mathcal{P}_{\mathrm{E}}$ principle $\pi^*(X)$ must be $\mathcal{P}_{\mathrm{E}}$ Pareto efficient to the original UMTSP (4.1).*

Proof. Suppose that $\pi^*(X)$ is the optimal route to single objective UTSP (4.3), but it is not the $\mathcal{P}_{\mathbb{E}}$ Pareto efficient route to the original UMTSP (3.1), by Definition 4.1, there must exist some $\bar{\pi}(X)$ such that $f_j(\bar{\pi}(X), \xi^{(j)}) \preceq f_j(\pi^*(X), \xi^j)$, and $f_j(\bar{\pi}(X), \xi^{(j)}) \prec f_j(\pi^*(X), \xi^j)$ for at least one index j, $(j = 1, 2, \ldots, p)$.

Without any loss of generality, let us assume when $j = j_0$, $f_{j_0}(\bar{\pi}(X), \xi_{j_0}) \prec f_{j_0}(\pi^*(X), \xi_{j_0})$, as $f_{j_0}^0$ is the lower bound of $f_{j_0}(\pi(X), \xi_{j_0})$, according to Theorem 4.3 we can get that

$$(f_j(\bar{\pi}(X), \xi^j) - f_j^0)^2 \prec (f_j(\pi^*(X), \xi^j) - f_j^0)^2$$

When $j \neq j_0$, according to Theorem 4.3 we can get that

$$(f_j(\bar{\pi}(X), \xi^j) - f_j^0)^2 \preceq (f_j(\pi^*(X), \xi^j) - f_j^0)^2$$

According to Theorem 4.2, we can obtain that

$$\sum_{j=1}^{p} (f_j(\bar{\pi}(X), \xi^j) - f_j^0)^2 \prec \sum_{j=1}^{p} (f_j(\pi^*(X), \xi^j) - f_j^0)^2$$

Then it follows from Theorem 4.4 that

$$\sqrt{\sum_{j=1}^{p} (f_j(\bar{\pi}(X), \xi^j) - f_j^0)^2} \prec \sqrt{\sum_{j=1}^{p} (f_j(\pi^*(X), \xi^j) - f_j^0)^2}$$

that is to say, $U(\bar{\pi}(X), \xi) \prec U(\pi^*(X), \xi)$. It follows from Definition 4.4 that $\pi^*(X)$ is not the optimal route to single objective UTSP (4.3), which contradicts with the previous hypothesis that $\pi^*(X)$ is the optimal route. Hence, $\pi^*(X)$ is $\mathcal{P}_{\mathbb{E}}$ Pareto efficient to the original UMTSP (3.1). The theorem is proved. □

Note that, in order to guarantee that the availability of Theorem 4.5, the lower bound of single objective $f_j(\pi(X), \xi^j)$ $(j = 1, 2, \ldots, p)$ on the feasible set, f_j^0, must exist. Actually, in the real-life TSP, nearly all optimization objectives are bounded, such as distance, cost, time, etc. Theorem 4.5 is also applicable to the situations as follows

$$U(\pi(X), \xi) = \left(\sum_{j=1}^{p} (f_j(\pi(X), \xi^j) - f_j^0)^q \right)^{1/q},$$

or

$$U(\pi(X), \xi) = \left(\sum_{j=1}^{p} \lambda_j (f_j(\pi(X), \xi^j) - f_j^0)^q \right)^{1/q}$$

where $\lambda \in \Lambda^{++} = \{\lambda = (\lambda_1, \ldots, \lambda_p)^T | \lambda_j > 0, \sum_{j=1}^{p} \lambda_j = 1\}$, q is integer greater than one.

5. ABC algorithm variation for TSP

Considering the uncertain, multiobjective and combinatorial characteristics in UMTSP, it is difficult to solve the problem effectively with the basic ABC algorithm. In this paper, an ABC algorithm variation inserted with the reverse operator, crossover operator and mutation operator in three bee phases is designed, which can solve UMTSP efficiently with the combination of *uncertain approach*.

5.1. The basic ABC algorithm

In the basic ABC algorithm, there are three essential components, that is, food source positions, nectar-amount, and three kinds of foraging bees (employed bees, onlookers, and scouts). Each food source position represents a feasible solution to the optimization problem being considered and its nectar-amount corresponds to the quality (fitness) of the solution. Each kind of foraging bee performs one particular operation for generating new candidate food source positions. Employed bees are those bees which are searching food around the food source in their memory currently, onlooker bees are those bees

Step 1: Send the employed bees to the food sources and determine the nectar amounts;
Step 2: Calculate the probability value of each food source;
Step 3: Send the onlooker bees to their food sources according to the probability values;
Step 4: Stop the exploitation process of the sources exhausted by the bees;
Step 5: Send the scouts into the search area for discovering new food sources randomly;
Step 6: Record the best food source found so far;
Step 7: If a stopping criterion is met, then output the best food source; otherwise, go back to *Step 1*.

Fig. 1. The procedure in basic ABC algorithm.

which are waiting in the hive for information from the employed bees, and scout bees are those bees which are carrying out random searches for discovering new food sources if the employed bees and onlookers cannot find a better neighboring food source. Thus, the ABC algorithm visualizes the employed and onlooker bees as performing the job of local search (exploitation), whereas the onlookers and scouts bees as performing the job of global search (exploration). Specifically, unlike real bee colonies, the ABC algorithm assumes that there is a one-to-one correspondence between the employed bees and the food sources, i.e., the number of food sources (solutions) is the same as the number of employed bees. The basic ABC algorithm follows an iterative process, which is shown in Fig. 1 (Karaboga, 2005).

5.2. Design of ABC algorithm variation

5.2.1. Population initialization

In our proposed algorithm, the route of TSP is represented as an integer permutation $\pi(X)$ as described in Section 4. Take a TSP with 14 cities for example, the integer permutation $(1, 4, 14, 2, 9, 3, 12, 8, 11, 13, 6, 5, 10, 7)$ is a feasible route for TSP. Based on the route representation, we can initialize the population with SN integer permutations randomly, where SN is the number of food sources.

5.2.2. Reverse operator

Usually employed and onlooker bees use the same search operators to perform exploitation search. In order to represent the behavior of exploitation search, the reverse operator is introduced, whose procedure is described as follows.

Step 1. Randomly generate two integers $r1$ and $r2$ from 1 to n (total number of locations).
Step 2. Set max$r$ as the maximum between $r1$ and $r2$ and min$r$ as the minimum between $r1$ and $r2$.
Step 3. Reverse the permutation order in selected solution from position min$r$ to position max$r$.

5.2.3. Crossover operator

In order to further improve the exploitation ability, the crossover operator is introduced to take the selected food source and the current optimal food source as parents to create better offspring, whose procedure is described as follows.

Step 1. Randomly generate two integers $r1$ and $r2$ from 1 to n (total number of locations).
Step 2. Set max$r$ as the maximum between $r1$ and $r2$ and min$r$ as the minimum between $r1$ and $r2$.
Step 3. Choose max$r$ and min$r$ as two crossing points, and then interchange the data between max$r$ and min$r$ in two parents.
Step 4. Delete the repeated number in offspring, and replace them through map relationship from the data interchanged in Step 3.

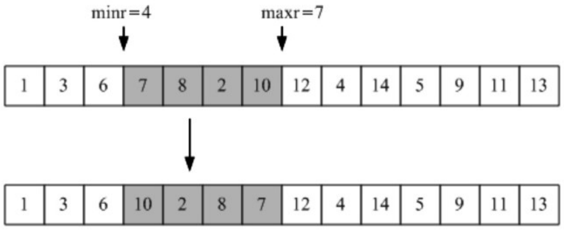

Fig. 2. Illustration of reverse operator for exploitation search.

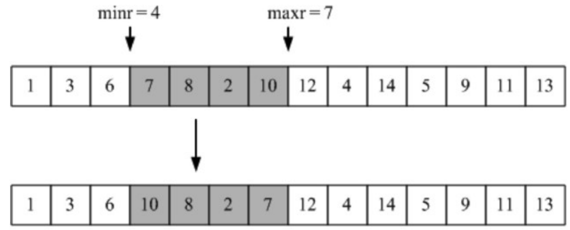

Fig. 4. Illustration of mutation operator for exploitation search.

5.2.4. Mutation operator

In order to improve the exploration ability, the mutation operator is applied to mutate the current best food source to discover new food source when the best food source does not change after certain cycles, whose procedure is described as follows.

Step 1. Randomly generate two integers $r1$ and $r2$ from 1 to n (total number of locations).

Step 2. Exchange the $r1$ position and $r2$ position in the selected solution.

The examples of reverse, crossover and mutation operator are illustrated in Figs. 2–4, where $r1$ and $r2$ are 4 and 7 respectively. Through the reverse operator, the employed bees can generate new neighboring food sources, and the onlooker bees can improve its local search ability by generating promising solutions around the selected solution. Through the crossover operator, the local search ability of employed and onlooker bees can be improved further. Through the mutation operator, the scout bees can generate new neighboring food sources around the current optimal solution to protect the search from getting in local minimums.

For the purpose of increasing the convergence speed and solution diversity, the reverse and crossover operator are conducted with reverse probability P_r and crossover probability P_c $(0 \le P_r, P_c \le 1)$. While since the mutation operator is conducted in scout bee phase, which is only activated if employed bees and onlookers cannot find a better neighboring food source within limited times rather than in every cycle, the mutation probability is actually relevant with the limited times and numbers of scout bees. So we do not consider the mutation probability in the design of algorithm, that is, set it as 1.

5.2.5. Employed bee phase

In this paper, the employed bees perform exploitation search through reverse and crossover operator around a given food source, and a greedy selection is applied between the current food source and its new neighboring food source to guarantee that better food sources will be reserved for further evolution. The greedy selection is based on the fitness of food source, which is calculated as follows:

$$\text{fit}_i = \begin{cases} 1/(1+f_i), & \text{if } f_i \ge 0 \\ 1+\text{abs}(f_i), & \text{if } f_i < 0 \end{cases}$$

where f_i is expected value of the uncertain single objective model described in model (4.1) or (4.3).

5.2.6. Onlooker bee phase

In this paper, the reverse operator and crossover operator are used in onlooker bee phase to generate new promising solutions around the food sources it chooses, which is according to a certain probability proportional to the chosen solution's fitness. The corresponding probability values of chosen solution can be calculated as follows:

$$p_i = \text{fit}_i \Big/ \sum_{i=1}^{SN} \text{fit}_i.$$

Just like the employed bee phase, a greedy selection is applied to guarantee the solution quality.

5.2.7. Scout bee phase

In this paper, the mutation operator is applied to implement the search around the current preferred solution if the employed bees and onlookers cannot find a better neighboring food source within limited times.

Straightforwardly, the framework of the proposed ABC algorithm is illustrated in Fig. 5. It can be seen that the proposed ABC algorithm not only applies reverse operator and crossover operator to generate new neighboring food sources, but also applies mutation operator to protect the search from getting in local minimums. It not only stresses the balance of global exploration and local exploitation, but also stresses the diversity of population during the searching process.

5.3. Investigation of parameter setting

Tuning algorithm parameters has significant importance on the performance of algorithm. In this subsection, the parameter setting of proposed ABC algorithm is investigated systematically based on

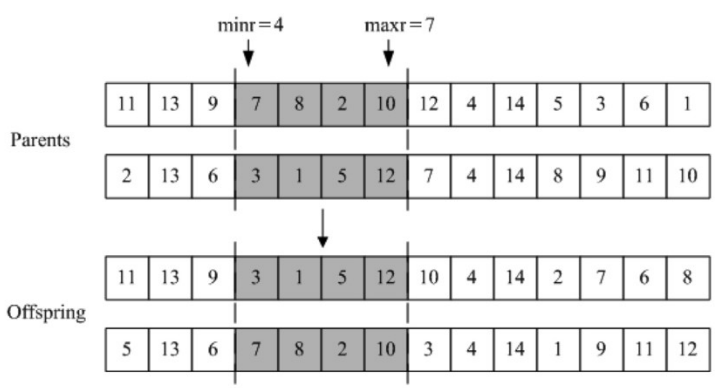

Fig. 3. Illustration of crossover operator for exploitation search.

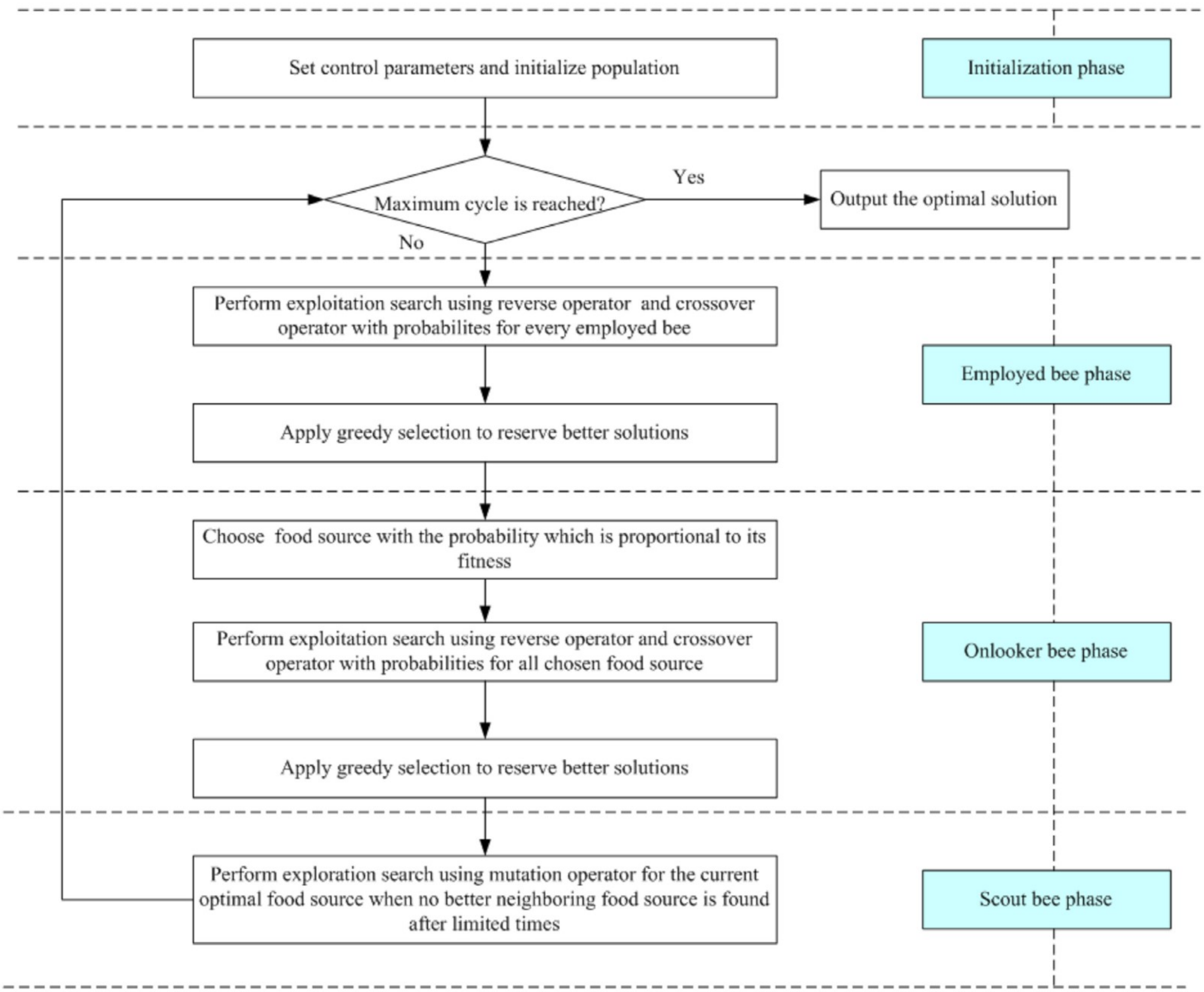

Fig. 5. Framework of proposed ABC for TSP.

Table 1
ABC algorithm's parameters and its levels considered.

Factors	Levels	Uncoded values		
		Low (1)	Medium (2)	High (3)
Combination of CS $*$ MCN	3	100*900	200*450	300*300
Crossover probability P_c	3	0.5	0.75	0.9
Reverse probability P_r	3	0.5	0.75	0.9
The value of limit	3	50	75	100

full factorial experimental design and analysis of variance (ANOVA). Table 1 shows the full factorial experimental design (3^k) adopted in this paper.

The first factor is the combination of the amount of colony size CS and the maximum cycle number MCN, which plays an important role on the amount of search in the solution space conducted by ABC algorithm. Higher values of both parameters can increase the probability of finding the best solutions but require longer computational time. In this paper, the amount of search (the combination of CS $*$ MCN) is predefined at 90,000. The second and third factors are the probability of crossover and reverse operator respectively, which can affect the convergence speed of ABC algorithm. The fourth factor is the limit value, which is the core parameter of the ABC algorithm dictating the occurrence of scout bees. Since the number of employed or onlookers bees is half of the colony size CS, we do not consider these two factors again. Furthermore, because the lower limit values can cause more scouts to be produced and higher limit values can cause scouts not to occur often, we only consider the value of limit in the parameter tuning and set the number of scout bees as 1.

Considering the scale of instance *st70.tsp* from TSPLIB (2012) is medium, the algorithm runs 10 times independently for each parameter combination to solve *st70.tsp* in MATLAB R2011a (version 7.12.0.635) on Intel(R) Core(TM) i3-2310M CPU @2.10GHz under Windows XP environment. The computational results obtained from 810 ($3^4 * 10$) runs are analyzed using a general linear model form of analysis of variance (ANOVA).

Table 2 shows the ANOVA table consisting of Source of Variation, Sum of Square, Degrees of Freedom, Mean Square, F value and P value. A factor with value of $P \leq 0.05$ is considered statistically significant with 95 percent confidence interval here. In Table 2, X_1, X_2, X_3 and X_4 denote the combination of CS $*$ MCN, crossover probability P_c, reverse probability P_r and the value of limit respectively. It can be seen that the combination of CS $*$ MCN, crossover probability P_c and reverse probability P_r are statistically significant while the value of limit and other interactions among four factors are found insignificant for the range considered.

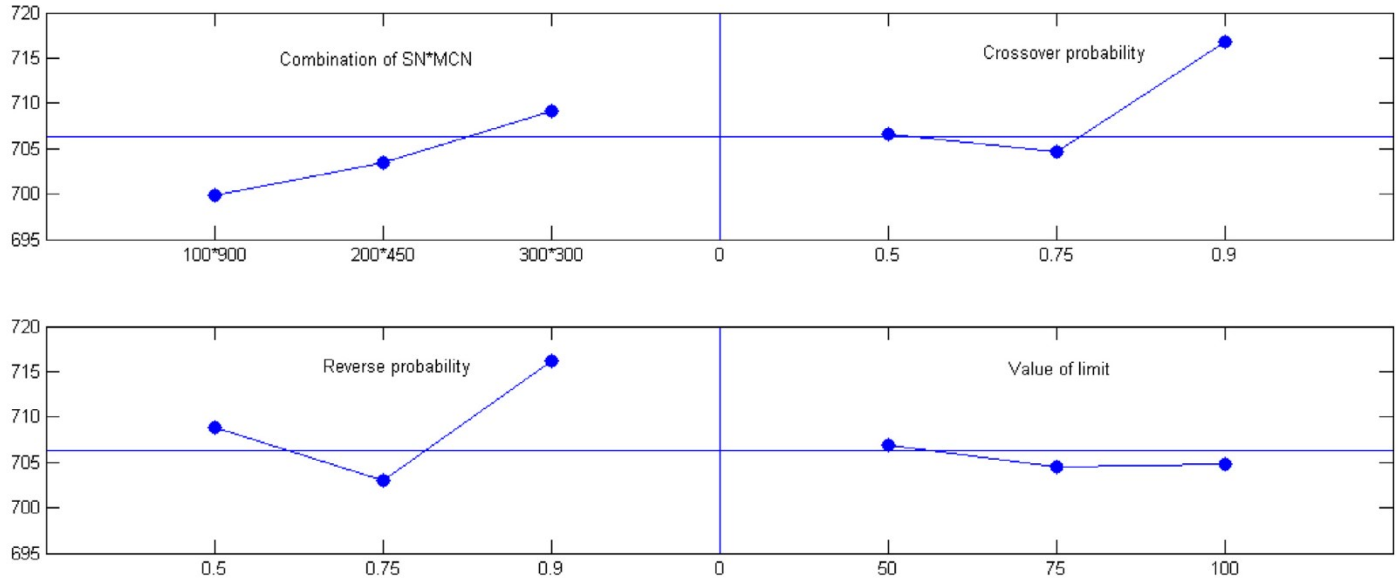

Fig. 6. Main effect plots on the ABC parameters.

Table 2
ANOVA on the ABC's parameters.

Source	Sum of square	d.f.	Mean square	F	Probability>F
X_1	3263.8	2	1631.9	3.66	0.0263
X_2	2794.6	2	1397.29	3.13	0.0442
X_3	3179.1	2	1589.57	3.56	0.0289
X_4	322.4	2	161.21	0.36	0.6969
X_1*X_2	2563.6	4	640.9	1.44	0.2202
X_1*X_3	1156.2	4	289.05	0.65	0.6286
X_1*X_4	3039.6	4	759.91	1.7	0.1474
X_2*X_3	2052	4	513.01	1.15	0.3319
X_2*X_4	2628.4	4	657.1	1.47	0.2087
X_3*X_4	1488.9	4	372.22	0.83	0.5036
X_1*X_2*X_3	3760.8	8	470.1	1.05	0.3941
X_1*X_2*X_4	4364.8	8	545.6	1.22	0.2825
X_1*X_3*X_4	3607.1	8	450.89	1.01	0.4264
X_2*X_3*X_4	3912.9	8	489.11	1.1	0.3609
X_1*X_2*X_3*X_4	4306.1	16	269.13	0.6	0.8828
Error	325276.9	729	446.2		
Total	366261.4	809			

Table 3
Parameter settings adopted in ABC algorithm.

Control parameters	Optimized	Un-optimized
Colony size	100	40
Maximum cycle number	900	2000
Crossover probability	0.75	1
Reverse probability	0.8	1
Limit	75	100
Number of onlooker bees	Half of the colony size	Half of the colony size
Number of employed bees	Half of the colony size	Half of the colony size
Number of scout bees	1	1

The main effect plots on the levels of the ABC's parameters against the computational results obtained from *st70.tsp* are illustrated in Fig. 6.

As shown in Fig. 6, the combination of CS ∗ MCN has large impact on the computational result. With limited amount of search at 90,000, the best result is achieved when the combination is set at 100*900. This indicates that the higher number of iterations is more preferable than the larger colony size under fixed search amount. The crossover and reverse probability, P_c and P_r, are performed best around the value of 0.75. The best condition of the value of limit is found best at the value between 75 and 100. The value of limit is the core parameter of the ABC algorithm dictating the occurrence of scout bees that are responsible for providing the diversity in the population. Since the crossover and reverse operator can provide sufficient diversity and improvement in the solution evolution, performance of the ABC algorithm does not change significantly when the limit values increased after certain value in this case. But low limit value may cause the population to be comprised of too many random solutions, so low values of the limit will worsen the performance of the ABC algorithm.

According to the analysis presented above, two parameter settings for the ABC algorithm are shown in Table 3, where one is optimized, another is un-optimized. And the box plot of the computation results obtained from the program with and without using optimized parameter setting is shown in Fig. 7. Each control parameter setting is computationally experimented with 40 replications.

It can be seen that the average computation result using optimized parameter setting is 691.6274 while 723.4573 is the average computation result using un-optimized parameter setting. This emphasizes that the ABC algorithm's performance can be improved significantly after adopting the optimized parameter setting.

5.4. Performance comparison with other algorithms

In order to test the performance of the proposed ABC algorithm in TSP, three benchmark symmetric TSPs from TSPLIB (2012) are utilized for experiment, including *dantzig42.tsp*, *st70.tsp* and *gr96.tsp*. At the same time, the distinguished algorithms for TSP, such as PSO, GA, ACO and SA, are used for comparison with the proposed ABC algorithm. Though these algorithms have also control parameters that should be tuned, analyzing the best parameter settings of these algorithms is out of our purpose and the corresponding parameters are recommended as follows.

In PSO, the number of individual is set at 300, the total number of iterations at 3000. In GA, the number of generation is set at 2000, the number of initial population at 50, selection probability at 0.9, crossover probability at 0.9, and mutation probability at 0.05. In ACO, the control parameters are set as $\alpha = 1$, $\beta = 5$, $\rho = 0.3$, $m = 50$, maxcycle $= 2000$, where α and β are used to control the relative weight of pheromone trail and heuristic value, and ρ is the pheromone trail decay coefficient, m is the number of ants, maxcycle is the total number of iterations. In SA, the initial temperature is set at 50,000, cooling rate at 0.95, and the terminal temperature at 0.001; considering that SA is not usually a population-based algorithm, the

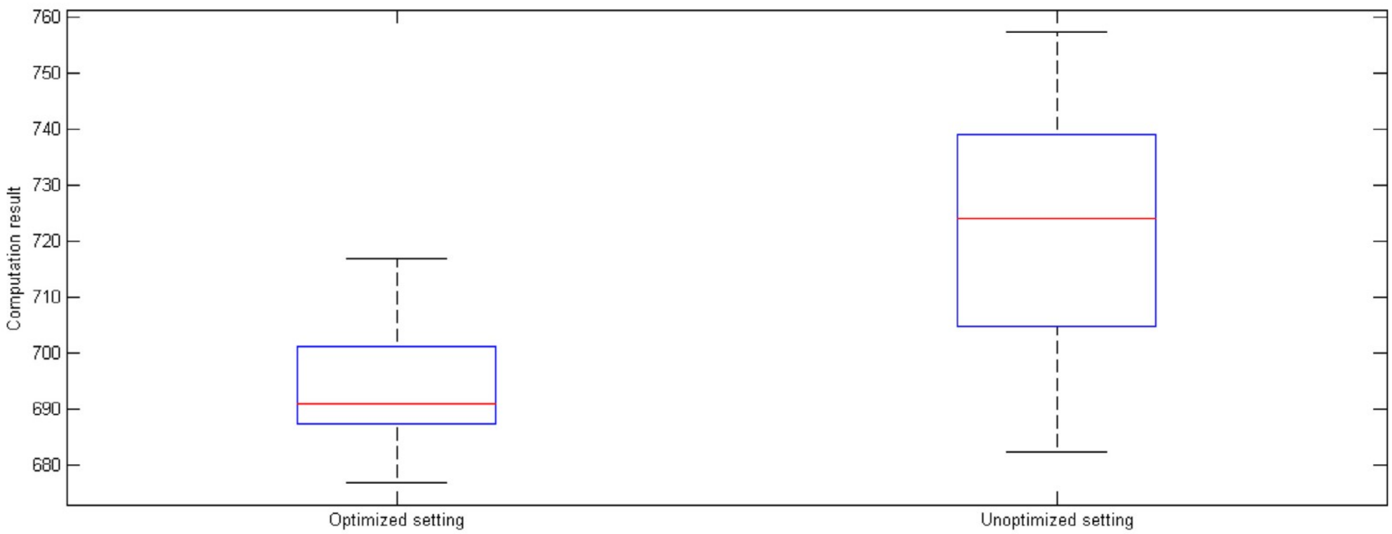

Fig. 7. Box plot of the computation results on two different parameter settings.

maximum iteration is extended especially for fairness. The threshold, or the total number of iterations in SA is set at 4000. The parameters used may not be the best combination. However, the parameters suggested were determined after preliminary testing considering both solution quality and computational efforts. In all of these algorithms, the update of best individual is realized through the mutation and crossover operator proposed in this paper. Furthermore, the basic ABC algorithm for TSP from Karaboga and Gorkemli (2011) is presented to compare with the proposed ABC algorithm.

In each case study, 40 independent runs of the algorithms with these parameters are carried out. Experiments are conducted in the same environment mentioned in Section 5.3. The computation results are presented in Table 4, where Min., Avg., Max., SD mean the minimum, average, maximum, and standard deviation of results obtained in 40 independent runs respectively, and Time is the average processing time cost in 40 independent runs. Furthermore, the average convergence figures of these algorithms on the three instances are presented in Figs. 8–10. Considering that each algorithm has different iterations, in order to compare the convergence speed among these algorithms, we predefined the number of iteration as 350.

As shown in Table 4 and Figs. 8–10, the proposed ABC algorithm outperforms other algorithms greatly, and can be competitive to the current optimal results. According to the good performances in the minimum, average, maximum and standard deviation values, it is concluded that the proposed ABC algorithm is of good searching quality and robustness. In addition, its average processing time is the lowest on all three instances, which further indicates that the proposed ABC algorithm is efficient and robust. Given the analogies between the solution of TSP and the UMTSP, it is reasonable to expect that the proposed ABC algorithm can achieve good performances in solving the UMTSP.

Table 4
Performance comparison on benchmark TSPs.

Instances		*danzig42.tsp*	*st70.tsp*	*gr96.tsp*
Optimal results published		679.2019	678.5975	512.3094
PSO	Min.	679.2019	677.1945	551.4460
	Avg.	699.8715	717.7294	574.3774
	Max.	722.9854	766.7405	608.7520
	SD	13.1413	22.7000	15.5858
	Time (s)	100.0650	185.8769	298.6339
GA	Min.	679.2019	692.4504	529.3467
	Avg.	715.8312	732.0563	558.2334
	Max.	762.0302	767.2600	583.1796
	SD	22.0551	21.3766	16.6326
	Time (s)	86.8691	102.0129	118.6581
ACO	Min.	703.8294	699.2357	539.4167
	Avg.	724.6758	710.3917	546.5933
	Max.	739.5636	715.4182	552.5655
	SD	10.5018	4.7699	3.5920
	Time (s)	334.2850	629.0889	916.2234
SA	Min.	686.2082	709.3605	589.1742
	Avg.	722.8525	781.5419	678.6547
	Max.	768.0446	878.0816	628.9470
	SD	22.6387	33.9260	28.7732
	Time (s)	69.9976	77.3834	83.0031
Basic ABC	Min.	679.2019	692.3274	530.9624
	Avg.	695.3776	712.0920	544.2715
	Max.	715.7774	739.3253	641.5035
	SD	12.1077	15.0072	23.6072
	Time (s)	92.5725	98.2874	112.3481
Proposed ABC	Min.	679.2019	677.1096	510.8863
	Avg.	685.3961	691.6318	527.8685
	Max.	701.1226	706.2756	548.3259
	SD	5.5500	8.8251	9.9122
	Time (s)	58.6272	71.5021	81.0858

6. Case study

Here, an application example is provided to illustrate the proposed solution approach of UMTSP. Since there currently exists no benchmark instances for UMTSP, the UMTSP case study presented is generated by extending the corresponding *ulysses16.tsp* from TSPLIB (2012).

6.1. Problem description

Consider the data in *ulysses16.tsp* as the coordinates of 16 locations in a transportation system, then we can obtain the deterministic distance matrix $D = (d_{ij})(n \times n)$, where d_{ij} means the distance between location i and j, and $d_{ij} = d_{ji}$. In order to extend *ulysses16.tsp* to a UMTSP, three objectives are considered, that is, travel time $T(\pi(X), \xi^{(1)})$, travel distance $D(\pi(X), \xi^{(2)})$, and travel cost $C(\pi(X), \xi^{(3)})$, where $\xi^{(1)} = (\xi_{ij}^{(1)})_{n\times n}$, $\xi^{(2)} = (\xi_{ij}^{(2)})_{n\times n}$, $\xi^{(3)} = (\xi_{ij}^{(3)})_{n\times n}$ mean the uncertain time matrix, uncertain distance matrix and uncertain cost matrix respectively. The uncertainty distribution of $\xi_{ij}^{(1)}$ is a zigzag uncertainty distribution $\Phi_{(1ij)}(x) = Z(2.8/d_{ij}, 6.7/d_{ij}, 10.4/d_{ij})$, the uncertainty distribution of $\xi_{ij}^{(2)}$ is a linear distribution $\Phi_{(2ij)}(x) = L(d_{ij}/2.3, d_{ij}/0.8)$, and the uncertainty distribution of $\xi_{ij}^{(3)}$ is a zigzag uncertainty distribution

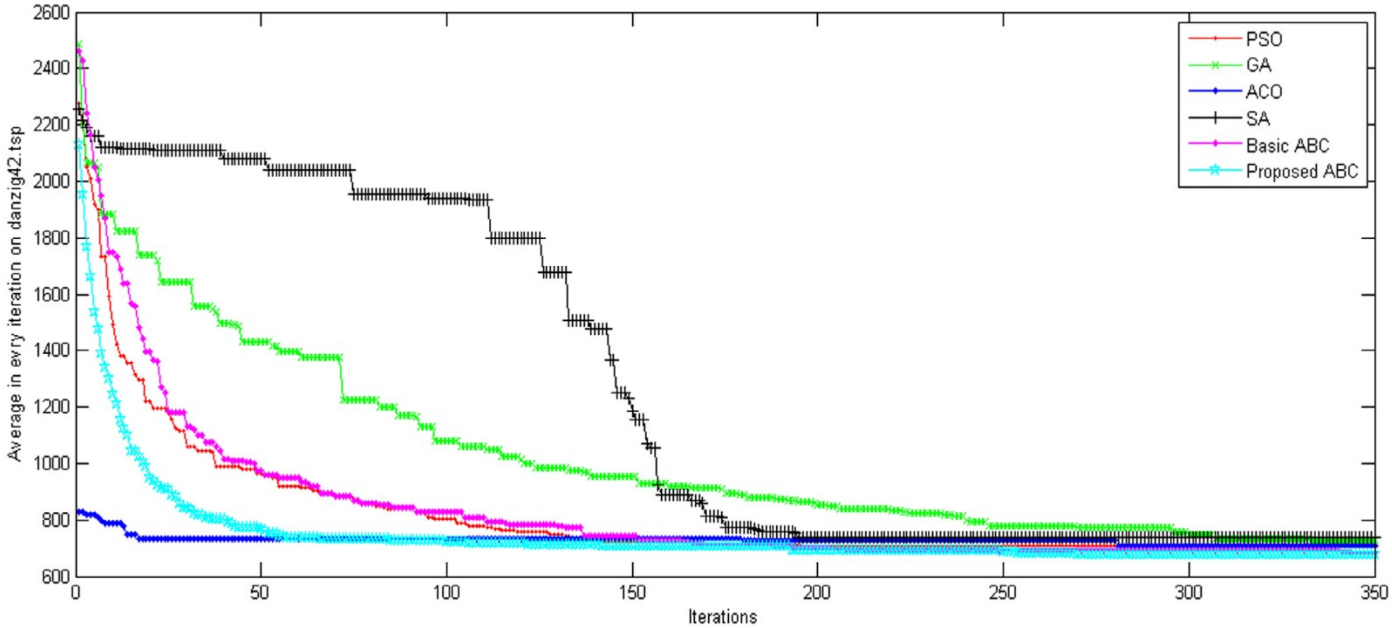

Fig. 8. Average convergence curve for *danzig42.tsp*.

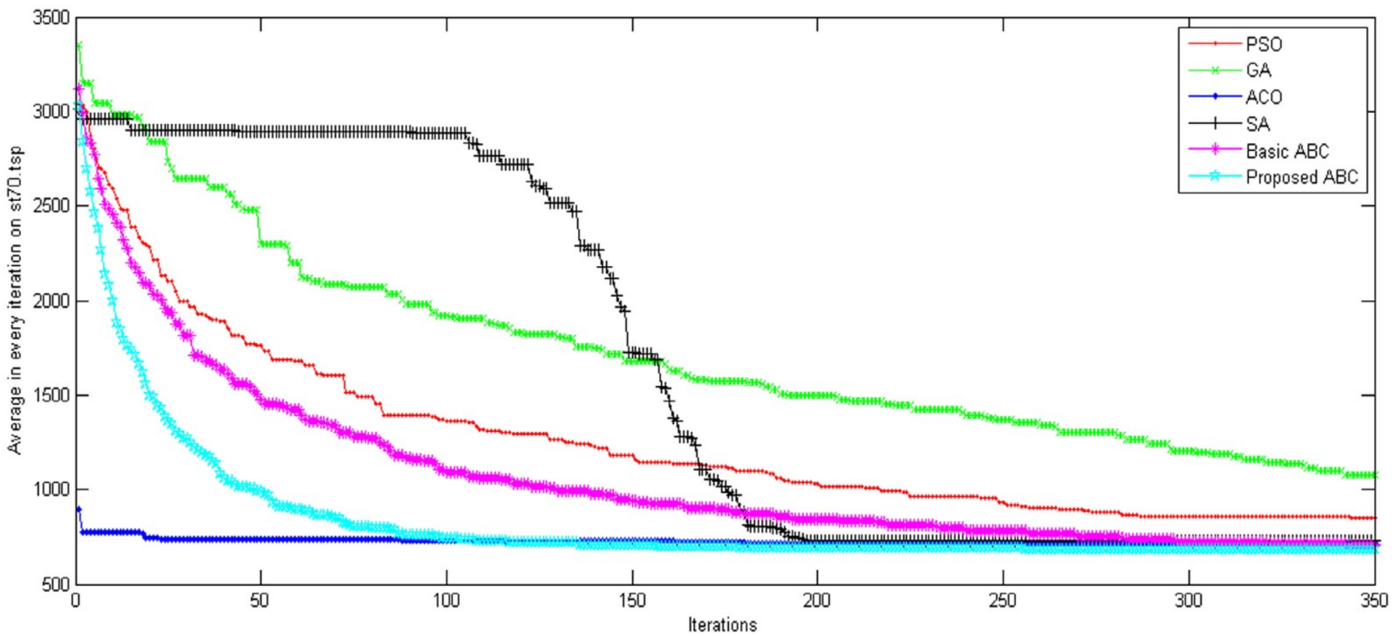

Fig. 9. Average convergence curve for *st70.tsp*.

$\Phi_{(3ij)}(x) = Z(|\ln(d_{ij}-5)^2| * 0.5, |\ln(d_{ij}-5)^2| * 2, |\ln(d_{ij}-5)^2| * 3)$. The components of $\xi^{(1)}$, $\xi^{(2)}$, $\xi^{(3)}$ are assumed to be independent with each other.

6.2. Mathematical formulation

According to the analysis in Section 3, the mathematical formulation of this case study can be presented as follows:

$$\begin{cases} \min\limits_{\pi(X)} T(\pi(X), \xi^{(1)}) = \sum_{i=1}^{n-1} \xi^{(1)}_{(K_i,K_{i+1})} + \xi^{(1)}_{(K_n,K_1)} \\ \min\limits_{\pi(X)} D(\pi(X), \xi^{(2)}) = \sum_{i=1}^{n-1} \xi^{(2)}_{(K_i,K_{i+1})} + \xi^{(2)}_{(K_n,K_1)} \\ \min\limits_{\pi(X)} C(\pi(X), \xi^{(3)}) = \sum_{i=1}^{n-1} \xi^{(3)}_{(K_i,K_{i+1})} + \xi^{(3)}_{(K_n,K_1)} \end{cases} \tag{6.1}$$

To solve problem (6.1), the ideal point method presented in Section 4 is applied. Firstly, we need to obtain the lower bound of three single objective functions. As they are all strictly increasing with respect to the uncertain variables contained, we set $\xi^{(1)} = 2.8/D$, $\xi^{(2)} = D/2.3$, $\xi^{(3)} = |\ln(D-5)^2| * 0.5$. The lower bound of three single objective functions can be denoted as T_0, D_0, and C_0. Then the UMTSP can be converted into formulation a single objective UTSP as follows

$$\min_{\pi(X)} U(\pi(X), \xi) = \sqrt{(T(\pi(X), \xi^{(1)}) - T_0)^2 + (D(\pi(X), \xi^{(2)}) - D_0)^2 + (C(\pi(X), \xi^{(3)}) - C_0)^2}$$

Since the three objective functions are all strictly increasing with respect to the uncertain variables contained, following Theorems 2.2 and 2.3, it is easy to obtain the equivalent deterministic TSP under $\mathcal{P}_E$ principle as follows

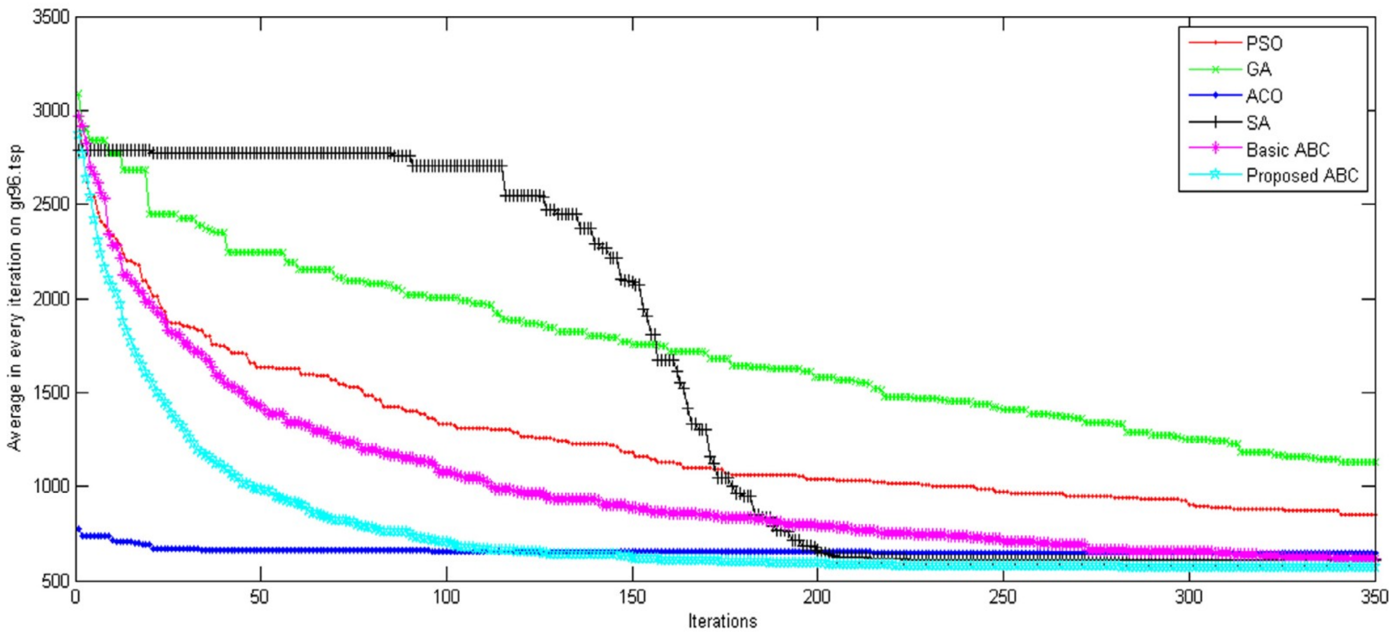

Fig. 10. Average convergence curve for *gr96.tsp*.

Table 5
Results obtained using uncertain approach.

Objectives	Results
T (Time)	16.4052
D (Distance)	102.7516
C (Cost)	60.3481
Optimal route	(4,14,5,7,11,6,10,15,16,3,1,2,8,12,9,13)

$$\min_{\pi(X)} E[U(\pi(X), \xi)]$$
$$= \int_0^1 \sqrt{(T(\pi(X), \Phi_{(1)}^{-1}(\alpha)) - T_0)^2 + (D(\pi(X), \Phi_{(2)}^{-1}(\alpha)) - D_0)^2 + (C(\pi(X), \Phi_{(3)}^{-1}(\alpha)) - C_0)^2}\, d\alpha$$

where $\Phi_{(k)}^{-1}(\alpha) = (\Phi_{(kij)}^{-1}(\alpha))_{n\times n}$, and $\Phi_{(kij)}^{-1}(\alpha)$ is the inverse uncertainty distribution of $\Phi_{(kij)}(x)$, $(k = 1, 2, 3)$.

6.3. Results and comments

After constructing the mathematical formulation, the proposed ABC algorithm is applied to obtain the optimal route. The optimized parameter setting shown in Table 3 is adopted. Firstly, the lower bound of three objective functions can be obtained as $T_0 = 4.4979$, $D_0 = 32.1685$, $C_0 = 31.6357$.

Then the result obtained using *uncertain approach* is shown in Table 5.

Specifically, the optimal route in original *ulysses16.tsp* is (9,11,5,15,8,4,2,3,1, 16,12,13,14,6,7,10), and corresponding objective values in UMTSP are $T = 58.4609$, $D = 62.3265$, $C = 303.4046$, respectively. It is shown that the optimal route in deterministic TSP with Euclidean distance can be a very poor route for other two objectives in the corresponding UMTSP with uncertain variables on the arc.

7. Conclusions

The general purpose of this study is to propose a novel TSP variation under uncertain environment with multiple objectives, called uncertain multiobjective TSP. A new solution approach of uncertain multiobjective optimization problem called *uncertain approach* is applied to solve UMTSP. To solve UMTSP efficiently, a new ABC algorithm is designed, which employs the reverse operator, crossover operator and mutation operator in three bee phases to improve the ability of global exploration and local exploitation. It is validated that the proposed ABC algorithm is competitive to other algorithms in the solution of classic TSP test problems. Results show that it is efficient to solve UMTSP with the combination of proposed ABC algorithm and *uncertain approach*. The construction and solution of UMTSP provide a new method in the solution of other uncertain combinatorial problems in practical application.

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